Algebra
Answer Booklet

Worked solutions, mark scheme and examiner tips for the Higher Tier workbook

For use by the tutor or teacher. Solutions show full working at the standard expected for full marks.

How to read the mark scheme

- **M1** is a method mark: given for a correct method even if there is an arithmetic slip. **A1** is an accuracy mark and is only given if the method mark was earned. **B1** is an independent mark for a correct statement, value or reason. **C1** is a communication mark for a clear concluding statement in a proof.
- Answers in **bold** are the final answers that must be seen. An answer with no working scores 0 on most multi-mark questions, so always insist on working.
- If a question asks for a particular accuracy (3 significant figures, 2 decimal places) the accuracy mark is lost if the accuracy is wrong. Rounding in the middle of a calculation is penalised.
- **Show that** and **prove** questions: every line of the working must be seen and the final line must match the result given in the question. Numerical examples are never a proof.
- Graph questions: allow the tolerances stated for each question; answers read from graphs are estimates. Sketches should show the shape, intercepts and turning points with coordinates.
- A grade guide (Grade 3 to Grade 9) is shown next to each question to help judge difficulty.

Marking approach

- Mark in a different colour pen. Write the score for each part beside the question and the total at the end of the section.
- For every lost mark, ask: was it knowledge (a formula or fact missing), method (wrong approach), accuracy (arithmetic or rounding) or presentation (no working, missing conclusion)? Record the category in the error log at the back.
- A student scoring 80% or more on grade 3 to 7 questions and at least half the marks on grade 8 to 9 questions is working at an approximate grade 8 to 9 standard for that topic.

1 Expanding and factorising: answers

41 marks

1.1 Expand and simplify $3(2x - 5) - 2(x - 4)$. **2 marks** | Grade 4

$6x - 15 - 2x + 8$ (M1)
$4x - 7$ (A1)

Examiner tip: The -2 multiplies both terms in its bracket, giving $-2x$ and $+8$.

1.2 Expand $4x(3x^2 - 2x + 5)$. **2 marks** | Grade 4

$12x^3$ (B1)
$12x^3 - 8x^2 + 20x$ (B1 for all three terms correct)

1.3 Expand and simplify $(x + 5)(x - 3)$. **2 marks** | Grade 5

$x^2 - 3x + 5x - 15$ (M1)
$x^2 + 2x - 15$ (A1)

1.4 Expand and simplify $(3x - 2)^2$. **2 marks** | Grade 5

$(3x - 2)(3x - 2) = 9x^2 - 6x - 6x + 4$ (M1)
$9x^2 - 12x + 4$ (A1)

Examiner tip: $9x^2 + 4$ or $9x^2 - 4$ scores no marks. Squaring a bracket always gives three terms.

1.5 Expand and simplify $(2x + 3)(5x - 4)$. **2 marks** | Grade 5

$10x^2 - 8x + 15x - 12$ (M1)
$10x^2 + 7x - 12$ (A1)

1.6 Factorise fully $8a^2b - 12ab^2$. **2 marks** | Grade 5

HCF is $4ab$ (B1 for 4, a or b factored out)
$4ab(2a - 3b)$ (B1)

1.7 Factorise **5 marks** | Grade 5

(a) $x^2 + 9x + 20$ *[2]*
$(x + 4)(x + 5)$ (M1 for numbers that multiply to 20, A1)
$(x + 4)(x + 5)$

(b) $x^2 - 5x - 24$ *[2]*
Numbers: -8 and $+3$ (M1)
$(x - 8)(x + 3)$ (A1)

(c) $y^2 - 64$ *[1]*
$(y - 8)(y + 8)$ (B1)

1.8 Factorise $3x^2 - 10x - 8$. **3 marks** | Grade 6

$ac = -24$; numbers -12 and $+2$ (M1)
$3x^2 - 12x + 2x - 8 = 3x(x - 4) + 2(x - 4)$ (M1)
$(3x + 2)(x - 4)$ (A1)

Examiner tip: Check by expanding: $3x^2 - 12x + 2x - 8 = 3x^2 - 10x - 8$.

1.9 Factorise fully $2x^2 - 50$. **3 marks** | Grade 6

Take out 2: $2(x^2 - 25)$ (M1)
$x^2 - 25 = (x - 5)(x + 5)$ (M1)
$2(x - 5)(x + 5)$ (A1)

Examiner tip: Always look for a common factor first. Stopping at $2(x^2 - 25)$ loses the final marks.

1.10 Expand and simplify $(x + 2)(x - 3)(x + 4)$. **3 marks** | Grade 7

$(x + 2)(x - 3) = x^2 - x - 6$ (M1)
$(x^2 - x - 6)(x + 4) = x^3 + 4x^2 - x^2 - 4x - 6x - 24$ (M1)
$x^3 + 3x^2 - 10x - 24$ (A1)

1.11 Simplify fully $(x^2 + x - 12) / (x^2 - 9)$. **3 marks** | Grade 7

Numerator $(x + 4)(x - 3)$ (M1)
Denominator $(x - 3)(x + 3)$ (M1)
$(x + 4)/(x + 3)$ (A1)

1.12 Write $3/(x + 2) + 4/(x - 1)$ as a single fraction. Give your answer in its simplest form. **3 marks** | Grade 7

Common denominator $(x + 2)(x - 1)$ (M1)
$[3(x - 1) + 4(x + 2)] / [(x + 2)(x - 1)]$ (M1)
$(7x + 5) / [(x + 2)(x - 1)]$ (A1)

1.13 Simplify fully $(2x^2 - 5x - 3) / (4x^2 - 1)$. **3 marks** | Grade 8

Numerator $(2x + 1)(x - 3)$ (M1)
Denominator $(2x - 1)(2x + 1)$ (M1)
$(x - 3)/(2x - 1)$ (A1)

Examiner tip: Spotting $4x^2 - 1$ as a difference of two squares is the key step.

1.14 Simplify $(x^2 - 4)/(x + 3) \times (x^2 + 3x)/(x - 2)$. **3 marks** | Grade 8

$x^2 - 4 = (x - 2)(x + 2)$ and $x^2 + 3x = x(x + 3)$ (M1)
Cancel $(x - 2)$ and $(x + 3)$ (M1)
$x(x + 2)$ (or $x^2 + 2x$) (A1)

1.15 Factorise fully $x^4 - 81$. **3 marks** | Grade 9

$x^4 - 81 = (x^2 - 9)(x^2 + 9)$ (M1)
$x^2 - 9 = (x - 3)(x + 3)$ (M1)
$(x - 3)(x + 3)(x^2 + 9)$ (A1)

Examiner tip: $x^2 + 9$ cannot be factorised, so the answer has three brackets.

2 Solving equations and inequalities: answers

43 marks

2.1 Solve $4x + 7 = 31$. **2 marks** | Grade 3

$4x = 24$ (M1)
$x = 6$ (A1)

2.2 Solve $7x - 3 = 4x + 15$. **2 marks** | Grade 4

$3x = 18$ (M1)
$x = 6$ (A1)

2.3 Solve $3(x + 4) = 2(x - 5)$. **3 marks** | Grade 5

$3x + 12 = 2x - 10$ (M1)
$3x - 2x = -10 - 12$ (M1)
$x = -22$ (A1)

2.4 Solve $(3x - 1)/4 = 5$. **2 marks** | Grade 5

$3x - 1 = 20$ (M1)
$x = 7$ (A1)

2.5 Solve $(x + 5)/3 + (x - 2)/2 = 7$. **3 marks** | Grade 6

Multiply by 6: $2(x + 5) + 3(x - 2) = 42$ (M1)
$5x + 4 = 42$ (M1)
$x = 7.6$ (A1)

2.6 Solve $6 - 2x \leq 14$. **2 marks** | Grade 5

$-2x \leq 8$ (M1)
$x \geq -4$ (A1)

Examiner tip: Dividing by −2 reverses the sign. This is the most common slip.

2.7 Write down all the integer values of x that satisfy $-3 < 2x + 1 \leq 9$. **3 marks** | Grade 6

$-4 < 2x \leq 8$ (M1)
$-2 < x \leq 4$ (A1)
−1, 0, 1, 2, 3, 4 (A1)

Examiner tip: −2 is not included because the left-hand inequality is strict.

2.8 The sides of a triangle are x cm, $(x + 4)$ cm and $(2x - 2)$ cm. The perimeter is 38 cm. Work out the length of the longest side. **3 marks** | Grade 6

$x + x + 4 + 2x - 2 = 38$ so $4x + 2 = 38$ (M1)
$x = 9$ (A1)
Longest side $= 2 \times 9 - 2 =$ **16 cm** (A1)

2.9 Solve $(2x + 1)/(x - 3) = 4$. **3 marks** | Grade 7

$2x + 1 = 4(x - 3)$ (M1)
$2x + 1 = 4x - 12$ (M1)
$x = 6.5$ (A1)

2.10 Solve $5/(x + 2) = 3/(x - 1)$. **3 marks** | Grade 7

$5(x - 1) = 3(x + 2)$ (M1)
$5x - 5 = 3x + 6$ (M1)
$x = 5.5$ (A1)

2.11 Solve $3x - 1 < 2x + 5 < 4x + 1$. **3 marks** | Grade 7

$3x - 1 < 2x + 5$ gives $x < 6$ (M1)
$2x + 5 < 4x + 1$ gives $4 < 2x$ so $x > 2$ (M1)
$2 < x < 6$ (A1)

2.12 Solve $(x - 2)/3 \geq (2x + 1)/5$. **3 marks** | Grade 7

Multiply by 15: $5(x - 2) \geq 3(2x + 1)$ (M1)
$5x - 10 \geq 6x + 3$ (M1)
$x \leq -13$ (A1)

Examiner tip: Moving x to the right-hand side avoids dividing by a negative. Moving it left gives $-x \geq 13$, which must be reversed.

2.13 Solve $x^2 - 3x - 10 \geq 0$. **3 marks** | Grade 7

$(x - 5)(x + 2) = 0$, critical values 5 and −2 (M1)
Sketch: U shape (M1)
$x \leq -2$ or $x \geq 5$ (A1)

2.14 Solve $2x^2 + x - 6 < 0$. **3 marks** | Grade 7

$(2x - 3)(x + 2) = 0$, critical values 1.5 and −2 (M1)
Sketch: U shape, below the axis between the roots (M1)
$-2 < x < 1.5$ (A1)

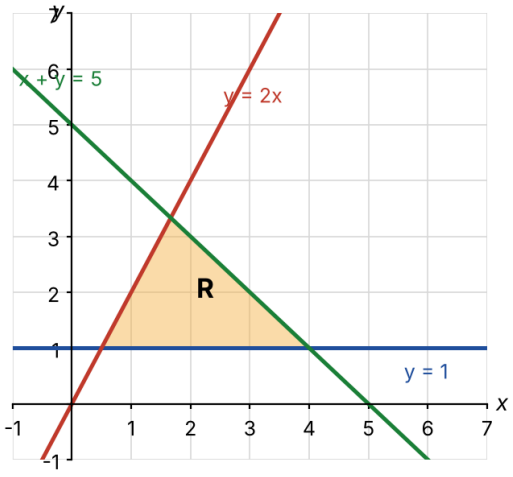
2.15 The lines $y = 1$, $y = 2x$ and $x + y = 5$ are drawn on the grid. The region R satisfies all three inequalities $y \geq 1$, $y \leq 2x$ and $x + y \leq 5$. **5 marks** | Grade 7

(a) On the grid, shade the region R and label it R. *[3]*
Correct side of $y = 1$ identified (above the line) (B1)
Correct sides of the other two lines identified (B1)
Triangle shaded and labelled R (B1)
Vertices at (0.5, 1), (4, 1) and (5/3, 10/3). See the answer diagram.

(b) Write down the coordinates of every point with integer coordinates that lies inside R or on its boundary. *[2]*
(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)
(M1 for at least 4 correct points, A1 for all 8 and no extras)

Examiner tip: Check each integer point in all three inequalities. The point (1, 3) fails because 3 is not ≤ 2 × 1.

Answer diagram


y
x
x + y = 5
y = 2x
R
y = 1
-1
1
2
3
4
5
6
7
-1
1
2
3
4
5
6

3 Quadratic equations and completing the square: answ

45 marks

3.1 Solve $x^2 + 7x + 10 = 0$. **3 marks** | Grade 5

(x + 5)(x + 2) = 0 (M1 for two correct factors)
x = −5 (A1)
x = −2 (A1)
x = −5 or x = −2

3.2 Solve $x^2 - 4x = 0$. **2 marks** | Grade 5

x(x − 4) = 0 (M1)
x = 0 or x = 4 (A1)

Examiner tip: Dividing by x loses the solution x = 0.

3.3 Solve $x^2 - 81 = 0$. **2 marks** | Grade 5

$x^2 = 81$ or (x − 9)(x + 9) = 0 (M1)
x = 9 or x = −9 (A1)

3.4 Solve $2x^2 + 5x - 3 = 0$. **3 marks** | Grade 6

(2x − 1)(x + 3) = 0 (M1 for method, A1 for correct factors)
x = 1/2 or x = −3 (A1)

3.5 Solve (x + 3)(x − 2) = 14. **3 marks** | Grade 6

$x^2 + x - 6 = 14$ so $x^2 + x - 20 = 0$ (M1)
(x + 5)(x − 4) = 0 (M1)
x = −5 or x = 4 (A1)

3.6 Solve $x^2 + 4x - 3 = 0$. Give your answers correct to 2 decimal places. **3 marks** | Grade 6

x = [−4 ± √(16 + 12)] / 2 (M1)
= −2 ± √7 (M1)
x = 0.65 or x = −4.65 (A1)

3.7 Solve $3x^2 - 8x - 2 = 0$. Give your answers correct to 2 decimal places. **3 marks** | Grade 7

x = [8 ± √(64 + 24)] / 6 (M1 for substitution into the formula)
= (8 ± √88) / 6 (M1)
x = 2.90 or x = −0.23 (A1)

Examiner tip: √88 = 9.3808. Do not round it before dividing.

3.8 **5 marks** | Grade 6

(a) Write $x^2 - 10x + 7$ in the form $(x + a)^2 + b$. *[2]*
$(x - 5)^2$ (M1)
$(x - 5)^2 - 18$ (A1)

(b) Hence solve $x^2 - 10x + 7 = 0$. Give your answers in the form $p \pm q\sqrt{2}$. *[2]*
$(x - 5)^2 = 18$ so $x - 5 = \pm\sqrt{18}$ (M1)
$\sqrt{18} = 3\sqrt{2}$, so **$x = 5 \pm 3\sqrt{2}$** (A1)

(c) Write down the coordinates of the turning point of the graph $y = x^2 - 10x + 7$. *[1]*
(5, −18) (B1)

3.9 Write $3x^2 + 12x + 5$ in the form $a(x + b)^2 + c$. **3 marks** | Grade 7

$3(x^2 + 4x) + 5$ (M1)
$3[(x + 2)^2 - 4] + 5$ (M1)
$3(x + 2)^2 - 7$ (A1)

Examiner tip: Check: $3(x + 2)^2 - 7 = 3x^2 + 12x + 12 - 7$.

3.10 The rectangle has length $(x + 5)$ cm and width $(x - 2)$ cm. The area of the rectangle is 18 cm^2. Work out the length and the width. **4 marks** | Grade 7

$(x + 5)(x - 2) = 18$ so $x^2 + 3x - 10 = 18$ (M1)
$x^2 + 3x - 28 = 0$ (M1)
$(x + 7)(x - 4) = 0$ so $x = 4$ ($x = -7$ rejected as a length would be negative) (A1)
Length **9 cm**, width **2 cm** (A1)

Examiner tip: Say why you reject the second root. Examiners look for it.

3.11 Show that $2x^2 - 3x + 5 = 0$ has no real solutions. **2 marks** | Grade 7

$b^2 - 4ac = 9 - 40$ (M1)
$= -31 < 0$, so there are no real roots (A1 with conclusion)

3.12 The equation $x^2 + kx + 9 = 0$ has equal roots. Find the possible values of k. **3 marks** | Grade 8

$b^2 - 4ac = 0$ (M1)
$k^2 - 36 = 0$ (M1)
$k = 6$ or $k = -6$ (A1)

3.13 Solve $3/(x + 2) + 2/(x - 1) = 1$. Give your answers correct to 2 decimal places. **5 marks** | Grade 8

$3(x - 1) + 2(x + 2) = (x + 2)(x - 1)$ (M1)
$5x + 1 = x^2 + x - 2$ (M1)
$x^2 - 4x - 3 = 0$ (A1)
$x = (4 \pm \sqrt{28}) / 2$ (M1)
$x = 4.65$ or $x = -0.65$ (A1)

3.14 The line $y = 2x + k$ is a tangent to the curve $y = x^2 + 4$. Work out the value of k. **4 marks** | Grade 9

$x^2 + 4 = 2x + k$ so $x^2 - 2x + (4 - k) = 0$ (M1)
A tangent touches once, so $b^2 - 4ac = 0$ (M1)
$4 - 4(4 - k) = 0$ (M1)
$k = 3$ (A1)

Examiner tip: Tangent means equal roots. Setting the discriminant to zero is the standard method.

4 Rearranging formulae: answers

43 marks

4.1 Make x the subject of y = 4x + 7. **2 marks** | Grade 4

4x = y − 7 (M1)
x = (y − 7)/4 (A1)

4.2 Make t the subject of v = u + at. **2 marks** | Grade 4

at = v − u (M1)
t = (v − u)/a (A1)

4.3 Make b the subject of A = ½(a + b)h. **3 marks** | Grade 5

2A = (a + b)h (M1)
2A/h = a + b (M1)
b = 2A/h − a (A1)

4.4 Make y the subject of 3(x + y) = 2y + 5x. **3 marks** | Grade 5

3x + 3y = 2y + 5x (M1)
3y − 2y = 5x − 3x (M1)
y = 2x (A1)

4.5 Make u the subject of $v^2 = u^2 + 2as$. **2 marks** | Grade 6

$u^2 = v^2 - 2as$ (M1)
$u = \sqrt{(v^2 - 2as)}$ (A1)

4.6 The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Make r the subject of the formula. **3 marks** | Grade 6

$3V = \pi r^2 h$ (M1)
$r^2 = 3V/(\pi h)$ (M1)
$r = \sqrt{(3V/(\pi h))}$ (A1)

4.7 Make x the subject of px + q = rx + s. **3 marks** | Grade 7

px − rx = s − q (M1)
x(p − r) = s − q (M1)
x = (s − q)/(p − r) (A1)

4.8 Make l the subject of $T = 2\pi\sqrt{(l/g)}$. **3 marks** | Grade 7

$T/(2\pi) = \sqrt{(l/g)}$ (M1)
$T^2/(4\pi^2) = l/g$ (M1)
$l = gT^2/(4\pi^2)$ (A1)

4.9 Make x the subject of $y = 3(x - 2)^2$. **3 marks** | Grade 7

$(x - 2)^2 = y/3$ (M1)
$x - 2 = \pm\sqrt{(y/3)}$ (M1)
$x = 2 \pm \sqrt{(y/3)}$ (A1)

Examiner tip: The ± is expected here because there is no context restricting the sign.

4.10 Make x the subject of $y = (2x + 3)/(x - 1)$. **4 marks** | Grade 8

$y(x - 1) = 2x + 3$ (M1)
$xy - y = 2x + 3$ (M1)
$xy - 2x = 3 + y$ so $x(y - 2) = y + 3$ (M1)
$x = (y + 3)/(y - 2)$ (A1)

4.11 **$1/R = 1/R_1 + 1/R_2$**. Make R_2 the subject of this formula. **3 marks** | Grade 8

$1/R_2 = 1/R - 1/R_1$ (M1)
$= (R_1 - R)/(R\,R_1)$ (M1)
$R_2 = R\,R_1/(R_1 - R)$ (A1)

4.12 $x = (a + b)/(a - b)$. Make a the subject of the formula. **4 marks** | Grade 8

$x(a - b) = a + b$ (M1)
$xa - xb = a + b$ (M1)
$xa - a = b + xb$ so $a(x - 1) = b(x + 1)$ (M1)
$a = b(x + 1)/(x - 1)$ (A1)

4.13 Show that if $y = 2x/(3 - x)$ then $x = 3y/(y + 2)$. **3 marks** | Grade 8

$y(3 - x) = 2x$ (M1)
$3y - xy = 2x$ so $3y = 2x + xy$ (M1)
$3y = x(2 + y)$ so **$x = 3y/(y + 2)$** (A1)

4.14 $q = \sqrt{[(p + 3)/(p - 1)]}$. Make p the subject of the formula. **5 marks** | Grade 9

$q^2 = (p + 3)/(p - 1)$ (M1)
$q^2(p - 1) = p + 3$ (M1)
$pq^2 - q^2 = p + 3$ (M1)
$pq^2 - p = q^2 + 3$ so $p(q^2 - 1) = q^2 + 3$ (M1)
$p = (q^2 + 3)/(q^2 - 1)$ (A1)

Examiner tip: Square first to remove the root, then treat it as a "subject appears twice" problem.

5 Sequences: answers

45 marks

5.1 Write down the next two terms of the sequence 7, 3, −1, −5, … **2 marks** | Grade 3

Common difference −4 (M1)
−9, −13 (A1)

5.2 Find the nth term of 7, 11, 15, 19, … **2 marks** | Grade 4

Common difference 4 (M1 for 4n)
4n + 3 (A1)

5.3 Find an expression for the nth term of 20, 17, 14, 11, … **2 marks** | Grade 5

Common difference −3, so −3n (M1)
23 − 3n (or −3n + 23) (A1)

Examiner tip: Decreasing sequences have a negative coefficient of n.

5.4 The nth term of a sequence is 5n − 2. **4 marks** | Grade 5

(a) Show that 148 is a term of the sequence. *[2]*
5n − 2 = 148 so 5n = 150 (M1)
n = 30, so 148 is the 30th term (A1)

(b) Is 200 a term of the sequence? Give a reason. *[2]*
5n − 2 = 200 gives n = 40.4 (M1)
No, because n is not a whole number (A1)

5.5 Here are the first three patterns of a sequence made from matchsticks. **5 marks** | Grade 5

(a) How many matchsticks are in pattern 4? *[1]*
13 (B1)

(b) Find an expression for the number of matchsticks in pattern n. *[2]*
Sequence 4, 7, 10 with difference 3 (M1)
3n + 1 (A1)

(c) Which pattern uses exactly 91 matchsticks? *[2]*
3n + 1 = 91 (M1)
n = 30, so **pattern 30** (A1)

5.6 The first two terms of a sequence are 3 and 4. Each term after that is the sum of the two previous terms. Find the 6th term. **2 marks** | Grade 5

3, 4, 7, 11, 18 (M1)
6th term = 11 + 18 = **29** (A1)

5.7 Here are the first four terms of a geometric sequence: 5, 10, 20, 40, … **3 marks** | Grade 6

(a) Write down the next two terms. *[1]*
80, 160 (B1)

(b) Work out the 10th term. *[2]*
5×2^9 (M1)
2560 (A1)

5.8 Find the nth term of the quadratic sequence 3, 9, 19, 33, 51, … **3 marks** | Grade 7

First differences 6, 10, 14, 18; second differences 4 (M1)
$2n^2$ = 2, 8, 18, 32, 50 (M1)
Remainder 1, 1, 1, 1, 1, so **$2n^2 + 1$** (A1)

5.9 Find the nth term of the quadratic sequence 6, 11, 18, 27, 38, … **3 marks** | Grade 7

First differences 5, 7, 9, 11; second differences 2 (M1)
n^2 = 1, 4, 9, 16, 25; remainder 5, 7, 9, 11, 13 (M1)
Remainder = 2n + 3, so **$n^2 + 2n + 3$** (A1)

5.10 Find the nth term of 12, 9, 4, −3, −12, … **3 marks** | Grade 7

First differences −3, −5, −7, −9; second differences −2 (M1)
$-n^2$ = −1, −4, −9, −16, −25; remainder 13, 13, 13, … (M1)
$13 - n^2$ (A1)

5.11 The diagrams show the first three patterns made from dots. **4 marks** | Grade 7

(a) Work out the number of dots in pattern 10. *[2]*
Dots 2, 6, 12 so nth term n(n + 1) (M1)
10 × 11 = **110** (A1)

(b) Which pattern has 420 dots? *[2]*
$n^2 + n = 420$ so $n^2 + n - 420 = 0$ (M1)
(n + 21)(n − 20) = 0 so **pattern 20** (A1)

5.12 The nth term of a sequence is $n^2 + 3n + 1$. **4 marks** | Grade 8

(a) Show that 55 is a term of the sequence. *[2]*
$n^2 + 3n + 1 = 55$ so $n^2 + 3n - 54 = 0$ (M1)
(n + 9)(n − 6) = 0 so n = 6 (A1). Check: 36 + 18 + 1 = 55

(b) Show that 60 is not a term of the sequence. *[2]*
$n^2 + 3n - 59 = 0$ (M1)
$b^2 - 4ac = 9 + 236 = 245$ is not a perfect square, so n is not a whole number (A1). Alternatively: 6th term is 55 and 7th term is 71, so 60 lies between them

Examiner tip: The "between two consecutive terms" argument is quicker: terms increase, 55 is the 6th term and 71 the 7th.

5.13 The nth term of a sequence is $3n^2 - 2n$. Show that the difference between the (n + 1)th term and the nth term is 6n + 1. **3 marks** | Grade 8

(n + 1)th term = $3(n + 1)^2 - 2(n + 1) = 3n^2 + 6n + 3 - 2n - 2$ (M1)
$= 3n^2 + 4n + 1$ (M1)
Subtract $3n^2 - 2n$: **6n + 1** (A1)

5.14 The first three terms of an arithmetic sequence are 2x + 1, 5x − 2 and 6x + 3. **5 marks** | Grade 8

(a) Work out the value of x. *[3]*
(5x − 2) − (2x + 1) = (6x + 3) − (5x − 2) (M1)
3x − 3 = x + 5 (M1)
x = 4 (A1)

(b) Find an expression for the nth term. *[2]*
Terms 9, 18, 27 (M1)
9n (A1)

6 Straight-line graphs: answers

45 marks

6.1 Work out the gradient of the straight line through (1, 3) and (5, 11). **2 marks** | Grade 4

(11 − 3)/(5 − 1) (M1)
2 (A1)

6.2 Line L is shown on the grid. Find an equation of L. **3 marks** | Grade 5

y-intercept 3 (B1)
Gradient = −3/6 = −½ (M1)
y = −½x + 3 (A1)

Examiner tip: Read the intercept from the y-axis. The line goes down, so the gradient is negative.

6.3 The equation of a line is 2y = 6x − 8. Write down the gradient and the y-intercept of the line. **2 marks** | Grade 5

y = 3x − 4 (M1)
Gradient **3**, y-intercept **−4** (A1)

6.4 Show that the point (3, 7) lies on the line 4x − y = 5. **2 marks** | Grade 5

Substitute x = 3, y = 7: 4 × 3 − 7 (M1)
= 5, so the point satisfies the equation (A1 with statement)

6.5 Find the coordinates of the point where the lines y = 2x − 1 and 3x + y = 14 intersect. **3 marks** | Grade 6

3x + 2x − 1 = 14 (M1)
5x = 15 so x = 3 (A1)
y = 2 × 3 − 1 = 5, so **(3, 5)** (A1)

6.6 Find an equation of the straight line through (2, 5) and (6, −3). **3 marks** | Grade 6

Gradient (−3 − 5)/(6 − 2) = −2 (M1)
5 = −2 × 2 + c (M1)
y = −2x + 9 (A1)

6.7 Find an equation of the line that is parallel to y = 3x − 1 and passes through (2, 4). **2 marks** | Grade 6

Gradient 3 (M1)
y = 3x − 2 (A1)

6.8 Line L_1 has the equation 3x + 2y = 12. Work out the gradient of L_1 and the coordinates of the points where it crosses the axes. **3 marks** | Grade 6

2y = −3x + 12 so y = −1.5x + 6, gradient **−1.5** (M1 A1)
Crosses the axes at **(0, 6)** and **(4, 0)** (B1)

6.9 A plumber charges a fixed call-out fee plus an hourly rate. A job lasting 1 hour costs £70. A job lasting 3 hours costs £130. **5 marks** | Grade 6

(a) Work out the hourly rate. *[2]*
(130 − 70)/(3 − 1) (M1)
£30 per hour (A1)

(b) Write a formula for the cost £C of a job lasting h hours. *[2]*
70 = 30 × 1 + call-out so call-out = £40 (M1)
C = 30h + 40 (A1)

(c) Work out the cost of a job that takes 5.5 hours. *[1]*
30 × 5.5 + 40 = **£205** (B1)

6.10 A is the point (−2, 3) and B is the point (4, −1). **5 marks** | Grade 7

(a) Work out the coordinates of the midpoint of AB. *[2]*
((−2 + 4)/2, (3 + (−1))/2) (M1)
(1, 1) (A1)

(b) Work out the length of AB. Give your answer in surd form. *[3]*
$AB^2 = 6^2 + 4^2 = 52$ (M1 M1)
√52 = 2√13 (A1)

6.11 The line A has the equation y = 2x + 1. P is the point (4, 1). Find an equation of the line through P that is perpendicular to A. **3 marks** | Grade 7

Perpendicular gradient −½ (M1)
1 = −½ × 4 + c (M1)
y = −½x + 3 (A1)

6.12 Find the equation of the line that passes through (0, −2) and is perpendicular to the line 2x + 3y = 6. **3 marks** | Grade 8

3y = −2x + 6 so gradient −⅔ (M1)
Perpendicular gradient 3/2 (M1)
y = 1.5x − 2 (A1)

6.13 A is the point (2, 1) and B is the point (8, 5). Find an equation of the perpendicular bisector of AB. **4 marks** | Grade 8

Midpoint (5, 3) (B1)
Gradient of AB = 4/6 = ⅔, so perpendicular gradient −1.5 (M1)
y − 3 = −1.5(x − 5) (M1)
y = −1.5x + 10.5 (A1)

6.14 The line L_1 has the equation y = 3x + 2. The line L_2 passes through A(0, 10) and is perpendicular to L_1. The lines meet at B. Work out the coordinates of B. **5 marks** | Grade 9

Gradient of L_2 = −⅓ so y = −⅓x + 10 (M1 A1)
3x + 2 = −⅓x + 10 (M1)
10x/3 = 8 so x = 2.4 (A1)
y = 3 × 2.4 + 2 = 9.2, so **B = (2.4, 9.2)** (A1)

7 Quadratic and other graphs: answers

46 marks

7.1 Here is a table of values for $y = x^2 - 3x$.

x = −1, 0, 1, 2, 3, 4
y = 4, ____, −2, −2, ____, 4

3 marks | Grade 4

(a) Complete the table. *[2]*
x = 0: y = 0 (B1)
x = 3: y = 9 − 9 = 0 (B1)

(b) Write down the equation of the line of symmetry of the graph. *[1]*
x = 1.5 (B1)

7.2 The graph of $y = x^2 - 2x - 3$ is shown.

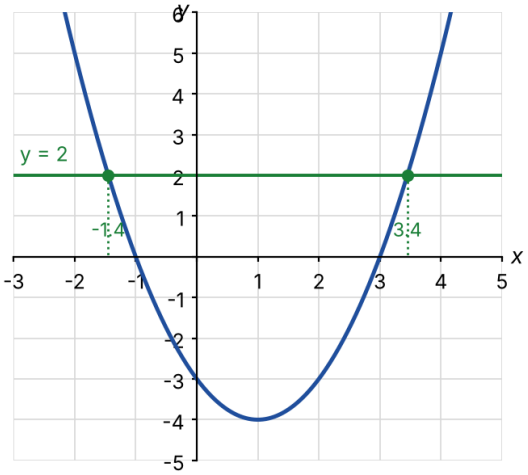
5 marks | Grade 5

(a) Write down the roots of $x^2 - 2x - 3 = 0$. *[1]*
x = −1 and x = 3 (B1)

(b) Write down the coordinates of the turning point. *[1]*
(1, −4) (B1)

(c) Use the graph to estimate the solutions of $x^2 - 2x - 3 = 2$. Show your method on the graph. *[3]*
Line y = 2 drawn (M1)
Read x-values where it meets the curve (M1)
x ≈ −1.4 and x ≈ 3.4 (A1) (accept −1.5 to −1.3 and 3.3 to 3.5)

Answer diagram



7.3 Write down the equation of the graph of each type described. Choose from $y = x^2$, $y = x^3$, $y = 1/x$ and $y = 2^x$.

4 marks | Grade 6

(a) A U-shaped curve, symmetrical about the y-axis. *[1]*
$y = x^2$ (B1)

(b) A curve with two separate branches that never touches either axis. *[1]*
y = 1/x (B1)

(c) A curve that passes through (0, 1) and (3, 8). *[1]*
$y = 2^x$ (B1)

(d) A curve that passes through (−1, −1) and (1, 1) and has rotational symmetry about the origin. *[1]*
$y = x^3$ (B1)

7.4 The graph of $y = a^x$ passes through (0, 1) and (3, 8). Find the value of a. **2 marks** | Grade 6

$a^3 = 8$ (M1)
a = 2 (A1)

7.5 For the graph $y = x^2 - 6x + 5$, work out the coordinates of the points where the graph crosses the axes and the coordinates of the turning point. Then sketch the graph. **4 marks** | Grade 6

$(x - 1)(x - 5) = 0$, so (1, 0) and (5, 0) (M1 A1)
y-intercept (0, 5) (B1)
Turning point at x = 3: $y = 9 - 18 + 5 = -4$, so **(3, −4)** (B1). Sketch: U shape through these points

7.6 Sketch the graph of $y = x(x - 2)(x + 1)$. Mark the points where the graph crosses the axes. **3 marks** | Grade 6

Correct cubic shape (positive x^3: bottom left to top right) (B1)
Crosses x-axis at −1, 0 and 2 (B1 B1)

7.7 Use the graph of y = sin x and y = cos x to find, for $0° \le x \le 360°$: **4 marks** | Grade 6

(a) two solutions of sin x = 0.5, given that sin 30° = 0.5 *[2]*
30° (B1) and **150°** (B1)

(b) two solutions of cos x = −0.5, given that cos 120° = −0.5 *[2]*
120° (B1) and **240°** (B1)

7.8 Show that the point (−3, 4) lies on the circle $x^2 + y^2 = 25$. Write down the equation of the circle with centre the origin and radius 7. **3 marks** | Grade 6

$(-3)^2 + 4^2 = 9 + 16 = 25$ (M1 A1)
$x^2 + y^2 = 49$ (B1)

7.9 The curve $y = x^2 + bx + c$ has a turning point at (3, −5). Find the values of b and c. **3 marks** | Grade 8

$y = (x - 3)^2 - 5$ (M1)
$= x^2 - 6x + 9 - 5$ (M1)
b = −6, c = 4 (A1)

7.10 Show that the curve $y = x^2 - 6x + 11$ is always above the x-axis. Write down the coordinates of its turning point. **3 marks** | Grade 8

$y = (x - 3)^2 + 2$ (M1)
$(x - 3)^2 \ge 0$, so $y \ge 2 > 0$ for all x (A1)
Turning point **(3, 2)** (B1)

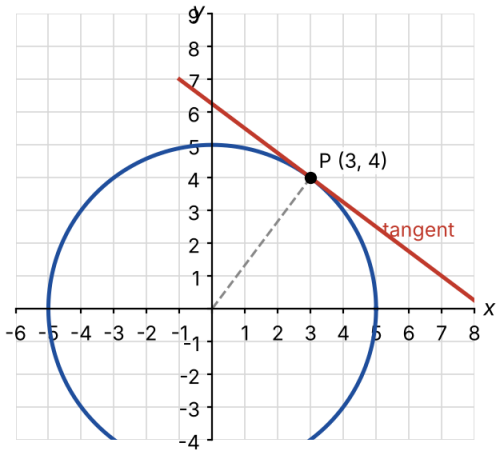
7.11 The graph of $y = k \times a^x$ passes through (0, 5) and (2, 45), and a > 0. Find k and a. **3 marks** | Grade 8

k = 5 (B1)
$5a^2 = 45$ so $a^2 = 9$ (M1)
a = 3 (A1)

7.12 The circle $x^2 + y^2 = 25$ is shown. P is the point (3, 4). Find an equation of the tangent to the circle at P. **4 marks** | Grade 8

Gradient of OP = 4/3 (B1)
Tangent gradient −3/4 (M1)
y − 4 = −¾(x − 3) (M1)
y = −¾x + 25/4 (3x + 4y = 25) (A1)

Answer diagram



7.13 The curve with equation $y = x^2$ passes through A(1, 1), B(3, 9), C(1.9, 3.61) and D(2.1, 4.41). **5 marks** | Grade 8

(a) Work out the gradient of the chord AB. *[1]*
(9 − 1)/(3 − 1) = **4** (B1)

(b) Work out the gradient of the chord CD. *[2]*
(4.41 − 3.61)/(2.1 − 1.9) (M1)
0.8/0.2 = **4** (A1)

(c) Write down an estimate for the gradient of the curve at the point (2, 4). Explain why your estimate is reliable. *[2]*
4 (B1)
CD is a chord with end points very close to (2, 4), so its gradient is almost the gradient of the tangent (B1)

8 Graph transformations: answers

57 marks

8.1 The graph of y = f(x) is shown. It has a maximum at (−1, 2) and a minimum at (1, −2). Write down the coordinates of the turning points of the graph of **8 marks** | Grade 6

(a) y = f(x) + 3 *[2]*
(−1, 5) and (1, 1) (B1 each)

(b) y = f(x + 2) *[2]*
(−3, 2) and (−1, −2) (B1 each)

(c) y = −f(x) *[2]*
(−1, −2) and (1, 2) (B1 each)

(d) y = f(−x) *[2]*
(1, 2) and (−1, −2) (B1 each)

Examiner tip: Test the rule with one point at a time. In f(−x) the x-coordinates change sign and the y-coordinates stay the same.

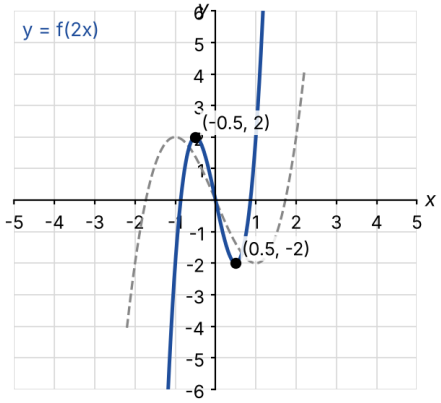
8.2 The graph of y = f(x) is shown. Write down the coordinates of the turning points of **6 marks** | Grade 7

(a) y = 2f(x) *[2]*
(−1, 4) and (1, −4) (B1 each)

(b) y = f(2x) *[2]*
(−0.5, 2) and (0.5, −2) (B1 each)

(c) y = f(x/2) *[2]*
(−2, 2) and (2, −2) (B1 each)

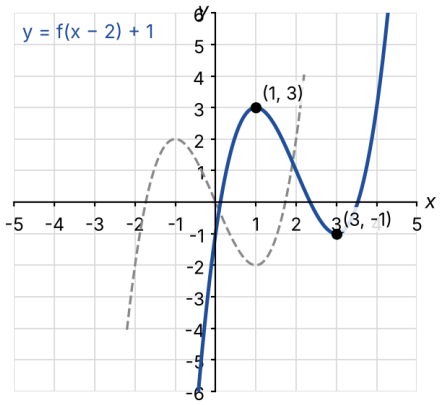
Answer diagram



8.3 On the grid the graph of y = f(x) is drawn. Sketch the graph of y = f(x − 2) + 1 and write the coordinates of its turning points. **3 marks** | Grade 7

Translation right 2 and up 1 (M1)
Turning points **(1, 3) and (3, −1)** (A1 A1)
Same shape, smooth curve, as in the answer diagram

Answer diagram



8.4 Describe fully the single transformation that maps the graph of y = f(x) onto the graph of **8 marks** | Grade 6

(a) y = f(x) + 7 *[2]*
Translation by vector **(0, 7)** (B1 B1)

(b) y = f(x − 5) *[2]*
Translation by vector **(5, 0)** (B1 B1)

(c) y = −f(x) *[2]*
Reflection in the x-axis (B1 B1)

(d) y = f(3x) *[2]*
Stretch parallel to the x-axis with **scale factor 1/3** (B1 B1)

8.5 The graph of $y = x^2$ is transformed. **5 marks** | Grade 7

(a) Write down the equation of the graph after a translation by the vector (3, 2). *[2]*
$(x - 3)^2$ (M1)
$y = (x - 3)^2 + 2$ (A1)

(b) Write down the turning point of the graph $y = (x + 4)^2 - 5$. *[1]*
(−4, −5) (B1)

(c) The graph of $y = x^2$ is reflected in the x-axis and then translated up by 6. Write down its equation. *[2]*
$y = -x^2$ (M1)
$y = -x^2 + 6$ (A1)

8.6 The graph of y = sin x is transformed. For each new graph, write down the maximum value, and for (c) the period. **4 marks** | Grade 7

(a) y = 3 sin x *[1]*
Maximum **3** (B1)

(b) y = sin x + 2 *[1]*
Maximum **3** (B1)

(c) y = sin 2x. Write down the period. *[1]*
180° (B1)

(d) y = cos x + 2. Write down the minimum value. *[1]*
cos x has minimum −1, so **1** (B1)

8.7 The graph of y = f(x) has a minimum at (2, −3). Write down the coordinates of the minimum of **4 marks** | Grade 7

(a) y = f(x) + 5 *[1]*
(2, 2) (B1)

(b) y = f(x − 1) *[1]*
(3, −3) (B1)

(c) y = 3f(x) *[1]*
(2, −9) (B1)

(d) y = f(2x) *[1]*
(1, −3) (B1)

8.8 The graph of y = f(x) has a maximum point at (4, 6). Write down the coordinates of the turning point of the graph of **7 marks** | Grade 8

(a) y = 2f(x) + 1 *[2]*
(4, 13) (M1 for 2 × 6, A1)

(b) y = f(x/2) *[2]*
(8, 6) (M1, A1)

(c) y = −f(x − 3). State whether it is a maximum or a minimum. *[3]*
x-coordinate 7 (B1); y-coordinate −6 (B1)
(7, −6) is a **minimum** (B1)

8.9 The curve $y = x^2 - 6x + 11$ is translated by the vector (−2, 3). Find the equation of the new curve in the form $y = x^2 + bx + c$. **3 marks** | Grade 8

$y = (x - 3)^2 + 2$, turning point (3, 2) (M1)
New turning point (1, 5), so $y = (x - 1)^2 + 5$ (M1)
$y = x^2 - 2x + 6$ (A1)

Examiner tip: A quicker route: replace x by (x + 2) in the equation and then add 3.

8.10 The graph of y = f(x) passes through the point (2, 5). Work out the coordinates of the point on y = 2f(x − 3) + 1 that corresponds to it. **3 marks** | Grade 8

x-coordinate: 2 + 3 = 5 (B1)
y-coordinate: 2 × 5 + 1 = 11 (M1)
(5, 11) (A1)

8.11 The graph of $y = x^2 - 4x + 7$ is reflected in the y-axis. Find the equation of the new graph. **3 marks** | Grade 8

Replace x with −x: $y = (-x)^2 - 4(-x) + 7$ (M1 M1)
$y = x^2 + 4x + 7$ (A1)

8.12 The curve with equation y = f(x) has a turning point at (a, b). The curve with equation y = f(3x) − 4 has a turning point at (6, 1). Work out a and b. **3 marks** | Grade 9

x-coordinate: a/3 = 6 so a = 18 (M1 A1)
y-coordinate: b − 4 = 1 so b = 5 (A1)
a = 18, b = 5

Examiner tip: Work backwards: undo the outside transformation on y and the inside transformation on x.

9 Simultaneous equations: answers

50 marks

9.1 Solve x + y = 10 and x − y = 4. **3 marks** | Grade 5

Add: 2x = 14 (M1)
x = 7 (A1)
y = 3 (A1)

9.2 Solve 3x + 2y = 12 and 5x − 2y = 4. **3 marks** | Grade 5

Add: 8x = 16 (M1)
x = 2 (A1)
y = 3 (A1)

9.3 Solve y = 3x − 2 and 2x + y = 13. **3 marks** | Grade 5

2x + 3x − 2 = 13 (M1)
x = 3 (A1)
y = 7 (A1)

9.4 Solve 2x + 3y = 7 and 3x + 5y = 11. **4 marks** | Grade 6

Multiply to equalise: 6x + 9y = 21 and 6x + 10y = 22 (M1)
Subtract: y = 1 (A1)
Substitute: 2x + 3 = 7 (M1)
x = 2, y = 1 (A1)

9.5 Solve 5x + 2y = 8 and 4x − 3y = 11. **4 marks** | Grade 6

15x + 6y = 24 and 8x − 6y = 22 (M1)
Add: 23x = 46 so x = 2 (M1 A1)
y = −1 (A1)
Examiner tip: Here the y-terms have opposite signs after scaling, so add.

9.6 Three adult tickets and two child tickets cost £34. Two adult tickets and five child tickets cost £41. Work out the cost of an adult ticket and the cost of a child ticket. **4 marks** | Grade 6

3a + 2c = 34 and 2a + 5c = 41 (M1)
15a + 10c = 170 and 4a + 10c = 82 (M1)
11a = 88 so a = 8 (A1)
Adult £8, child £5 (A1)

9.7 Three pens and four pencils cost £2.90. Five pens and two pencils cost £3.20. Work out the cost of one pen and the cost of one pencil. **4 marks** | Grade 6

3p + 4q = 2.90 and 5p + 2q = 3.20 (M1)
10p + 4q = 6.40 (M1)
Subtract: 7p = 3.50 so p = 0.50 (A1)
Pen 50p, pencil 35p (A1)

9.8 Solve $y = x + 2$ and $y = x^2 - 4$. **4 marks** | Grade 7

$x + 2 = x^2 - 4$ so $x^2 - x - 6 = 0$ (M1)
$(x - 3)(x + 2) = 0$ (M1)
$x = 3$, $y = 5$ (A1)
$x = -2$, $y = 0$ (A1)

9.9 Solve $y = x^2 - 3x$ and $y = 2x - 6$. **4 marks** | Grade 7

$x^2 - 3x = 2x - 6$ so $x^2 - 5x + 6 = 0$ (M1)
$(x - 2)(x - 3) = 0$ (M1)
$x = 2$, $y = -2$ (A1)
$x = 3$, $y = 0$ (A1)

9.10 The circle $x^2 + y^2 = 25$ and the line $y = x + 1$ are shown. Solve the simultaneous equations $x^2 + y^2 = 25$ and $y = x + 1$ algebraically. **5 marks** | Grade 8

$x^2 + (x + 1)^2 = 25$ (M1)
$2x^2 + 2x - 24 = 0$ so $x^2 + x - 12 = 0$ (M1 A1)
$(x + 4)(x - 3) = 0$ (M1)
(3, 4) and (−4, −3) (A1)

9.11 Solve the simultaneous equations $x^2 + y^2 = 10$ and $y = 3x$. **4 marks** | Grade 8

$x^2 + 9x^2 = 10$ (M1)
$x^2 = 1$ so $x = \pm 1$ (M1 A1)
(1, 3) and (−1, −3) (A1)

9.12 Show that the line $y = x + 5$ is a tangent to the curve $y = x^2 - x + 6$. **3 marks** | Grade 8

$x^2 - x + 6 = x + 5$ so $x^2 - 2x + 1 = 0$ (M1)
$(x - 1)^2 = 0$ or $b^2 - 4ac = 4 - 4 = 0$ (M1)
One repeated root, so the line touches the curve at only one point (1, 6): it is a tangent (A1)

9.13 The line $y = 2x + k$ is a tangent to the circle $x^2 + y^2 = 20$. Find the two possible values of k. **5 marks** | Grade 9

$x^2 + (2x + k)^2 = 20$ (M1)
$5x^2 + 4kx + k^2 - 20 = 0$ (M1)
Tangent: $b^2 - 4ac = 0$, $16k^2 - 20(k^2 - 20) = 0$ (M1)
$-4k^2 + 400 = 0$ so $k^2 = 100$ (A1)
$k = 10$ or $k = -10$ (A1)

Examiner tip: Substitute, form a quadratic in x, then set the discriminant to 0 for a tangent.

10 Algebraic proof: answers

45 marks

10.1 n is an integer. Write down an expression for **3 marks** | Grade 5

(a) an even number *[1]*
2n (B1)

(b) the next odd number after 2n + 1 *[1]*
2n + 3 (B1)

(c) the sum of three consecutive integers n, n + 1 and n + 2, in its simplest form *[1]*
3n + 3 (B1)

10.2 Prove that the sum of two consecutive integers is always odd. **3 marks** | Grade 6

n and n + 1 (B1)
n + n + 1 = 2n + 1 (M1)
2n is even so 2n + 1 is odd (C1)

10.3 Prove that the sum of three consecutive integers is a multiple of 3. **3 marks** | Grade 6

n, n + 1, n + 2 (B1)
3n + 3 (M1)
= 3(n + 1), a multiple of 3 (C1)

10.4 Prove that the sum of any two consecutive odd numbers is a multiple of 4. **3 marks** | Grade 7

2n + 1 and 2n + 3 (B1)
4n + 4 (M1)
= 4(n + 1), a multiple of 4 (C1)

10.5 **4 marks** | Grade 7

(a) Show that $(n + 3)^2 - (n - 3)^2 = 12n$. *[3]*
$n^2 + 6n + 9$ (M1)
$n^2 - 6n + 9$ (M1)
Subtract: **12n** (A1)

(b) Explain why $(n + 3)^2 - (n - 3)^2$ is a multiple of 12 for every integer n. *[1]*
12n = 12 × n, an integer multiple of 12 (C1)

10.6 Prove that the product of an odd number and an even number is always even. **3 marks** | Grade 7

Odd 2a + 1, even 2b (B1)
(2a + 1) × 2b = 2b(2a + 1) (M1)
2 × an integer, so even (C1)

Examiner tip: Use different letters (a and b). Using n for both would only prove the result for equal numbers.

10.7 Prove that the sum of the squares of two consecutive integers is always odd. **3 marks** | Grade 7

$n^2 + (n + 1)^2 = 2n^2 + 2n + 1$ (M1 M1)
$= 2(n^2 + n) + 1$, which is odd (C1)

10.8 **3 marks** | Grade 7

(a) Write down an example to show that the sum of two prime numbers is not always even. *[1]*
For example 2 + 3 = 5 (B1)

(b) Jon says "$n^2 + n + 41$ is always a prime number". Show that Jon is wrong. *[2]*
Try n = 40: 1600 + 40 + 41 = 1681 (M1)
1681 = 41 × 41, not prime (A1)

10.9 Prove that the difference between the squares of any two consecutive odd numbers is a multiple of 8. **3 marks** | Grade 8

$(2n + 3)^2 - (2n + 1)^2$ (B1)
$= (4n^2 + 12n + 9) - (4n^2 + 4n + 1) = 8n + 8$ (M1)
$= 8(n + 1)$, a multiple of 8 (C1)

10.10 Prove that the square of any odd number is 1 more than a multiple of 8. **4 marks** | Grade 8

$(2n + 1)^2 = 4n^2 + 4n + 1$ (M1)
$= 4n(n + 1) + 1$ (M1)
$n(n + 1)$ is the product of consecutive integers so one factor is even (B1)
4 × even is a multiple of 8, so the expression is 1 more than a multiple of 8 (C1)

10.11 Show that $1/n - 1/(n + 1) = 1/[n(n + 1)]$. **3 marks** | Grade 8

Common denominator $n(n + 1)$ (M1)
$[(n + 1) - n] / [n(n + 1)]$ (M1)
$= 1/[n(n + 1)]$ (A1)

10.12 Show that $(2n + 3)^2 - (2n - 1)^2$ is always a multiple of 8. **3 marks** | Grade 8

$[(2n + 3) - (2n - 1)][(2n + 3) + (2n - 1)]$ (M1)
$= 4(4n + 2) = 16n + 8$ (M1)
$= 8(2n + 1)$, a multiple of 8 (C1)

10.13 Prove that the sum of the squares of any three consecutive integers is never a multiple of 3. **4 marks** | Grade 9

$n^2 + (n + 1)^2 + (n + 2)^2$ (B1)
$= 3n^2 + 6n + 5$ (M1)
$= 3(n^2 + 2n + 1) + 2$ (M1)
This is 2 more than a multiple of 3, so it is never a multiple of 3 (C1)

10.14 Prove that $x^2 + y^2 \geq 2xy$ for all real numbers x and y. **3 marks** | Grade 9

$(x - y)^2 \geq 0$ because a square cannot be negative (B1)
$x^2 - 2xy + y^2 \geq 0$ (M1)
$x^2 + y^2 \geq 2xy$ (C1)

11 Functions: answers

46 marks

11.1 f(x) = 4x − 7. **4 marks** | Grade 4

(a) Work out f(5). *[1]*
13 (B1)

(b) Work out f(−2). *[1]*
4(−2) − 7 = **−15** (B1)

(c) Solve f(x) = 21. *[2]*
4x − 7 = 21 (M1)
x = 7 (A1)

11.2 $g(x) = x^2 + 3x$. **5 marks** | Grade 6

(a) Work out g(4) and g(−3). *[2]*
g(4) = 16 + 12 = **28** (B1)
g(−3) = 9 − 9 = **0** (B1)

(b) Solve g(x) = 10. *[3]*
$x^2 + 3x - 10 = 0$ (M1)
(x + 5)(x − 2) = 0 (M1)
x = 2 or x = −5 (A1)

11.3 f(x) = 2x + 3 and $g(x) = x^2$. Find, in their simplest forms, **4 marks** | Grade 6

(a) fg(x) *[2]*
$f(x^2)$ (M1)
$2x^2 + 3$ (A1)

(b) gf(x) *[2]*
$(2x + 3)^2$ (M1)
$4x^2 + 12x + 9$ (A1)

11.4 $f(x) = 3x - 4$. Find $f^{-1}(x)$. **2 marks** | Grade 6

y = 3x − 4, so x = (y + 4)/3 (M1)
$f^{-1}(x) = (x + 4)/3$ (A1)

11.5 $f(x) = (x - 2)/5$. Find $f^{-1}(x)$. **2 marks** | Grade 6

x = 5y + 2 (M1)
$f^{-1}(x) = 5x + 2$ (A1)

11.6 f(x) = x + 5 and g(x) = 3x. Show that fg(x) − gf(x) = −10. **3 marks** | Grade 7

fg(x) = f(3x) = 3x + 5 (M1)
gf(x) = g(x + 5) = 3(x + 5) = 3x + 15 (M1)
3x + 5 − (3x + 15) = **−10** (A1)

11.7 $f(x) = (3x + 1)/(x - 2)$. **5 marks** | Grade 8

(a) Write down the value of x that cannot be used in this function. *[1]*
x = 2 (B1)

(b) Find $f^{-1}(x)$. *[4]*
$y(x - 2) = 3x + 1$ (M1)
$xy - 2y = 3x + 1$ (M1)
$x(y - 3) = 1 + 2y$ (M1)
$f^{-1}(x) = (2x + 1)/(x - 3)$ (A1)

11.8 $f(x) = 2x - 1$ and $g(x) = x^2$. Solve $fg(x) = gf(x)$. **4 marks** | Grade 8

$fg(x) = 2x^2 - 1$ (B1)
$gf(x) = (2x - 1)^2 = 4x^2 - 4x + 1$ (B1)
$2x^2 - 4x + 2 = 0$ (M1)
x = 1 (A1)

11.9 $f(x) = x^2 - 4x$ and $g(x) = x + 1$. Solve $fg(x) = 0$. **4 marks** | Grade 8

$fg(x) = (x + 1)^2 - 4(x + 1)$ (M1)
$= x^2 - 2x - 3$ (A1)
$(x - 3)(x + 1) = 0$ (M1)
x = 3 or x = −1 (A1)

11.10 $f(x) = \sqrt{(x - 3)}$. **5 marks** | Grade 8

(a) Write down the domain of f. *[1]*
x ≥ 3 (B1)

(b) Solve $f(x) = 5$. *[2]*
$x - 3 = 25$ (M1)
x = 28 (A1)

(c) Find $f^{-1}(x)$. *[2]*
$x = y^2 + 3$ (M1)
$f^{-1}(x) = x^2 + 3$ (A1)

11.11 $f(x) = ax + b$. Given that $f(2) = 7$ and $f(-1) = -2$, find a and b and write down $f^{-1}(x)$. **4 marks** | Grade 8

$2a + b = 7$ and $-a + b = -2$ (M1)
$3a = 9$ so $a = 3$ (M1 A1)
$b = 1$, $f(x) = 3x + 1$ and **$f^{-1}(x) = (x - 1)/3$** (A1)

11.12 $f(x) = (x + 3)/(x - 1)$ for $x \neq 1$. Show that f is its own inverse, that is $ff(x) = x$. **4 marks** | Grade 9

$ff(x) = [(x + 3)/(x - 1) + 3] \div [(x + 3)/(x - 1) - 1]$ (M1)
Multiply top and bottom by $(x - 1)$: $(x + 3 + 3x - 3)/(x + 3 - x + 1)$ (M1 M1)
$= 4x/4 =$ **x** (A1)

Examiner tip: Multiplying numerator and denominator by $(x - 1)$ clears the inner fractions in one step.

12 Iteration: answers

43 marks

12.1 $x_{n+1} = 2x_n - 3$ and $x_1 = 5$. Work out x_2, x_3 and x_4. **3 marks** | Grade 5

$x_2 = 2 \times 5 - 3 =$ **7** (B1)
$x_3 =$ **11** (B1)
$x_4 =$ **19** (B1)

12.2 Show that the equation $x^3 + x - 5 = 0$ has a solution between x = 1 and x = 2. **2 marks** | Grade 6

f(1) = 1 + 1 − 5 = −3 and f(2) = 8 + 2 − 5 = 5 (M1)
Change of sign, so a root lies between 1 and 2 (A1 with conclusion)

12.3 $x^3 + x - 5 = 0$ has a solution between 1 and 2. **6 marks** | Grade 6

(a) Show that the equation can be rearranged to give $x = \sqrt[3]{(5 - x)}$. *[2]*
$x^3 = 5 - x$ (M1)
Take cube root: $x = \sqrt[3]{(5 - x)}$ (A1)

(b) Use $x_{n+1} = \sqrt[3]{(5 - x_n)}$ with $x_0 = 1.5$ to find x_1, x_2 and x_3. Give each answer to 4 decimal places. *[3]*
$x_1 = 1.5183$ (B1)
$x_2 = 1.5156$ (B1)
$x_3 = 1.5160$ (B1)

(c) Write down the solution of the equation to 2 decimal places. *[1]*
1.52 (B1)

12.4 $x_{n+1} = \sqrt{(2x_n + 3)}$ and $x_0 = 2$. **5 marks** | Grade 6

(a) Work out x_1, x_2 and x_3 to 4 decimal places. *[3]*
$x_1 = 2.6458$ (B1)
$x_2 = 2.8795$ (B1)
$x_3 = 2.9596$ (B1)

(b) The sequence approaches the solution of $x^2 = 2x + 3$. What is this solution (the one it approaches)? *[2]*
$x^2 - 2x - 3 = 0$ so $(x - 3)(x + 1) = 0$ (M1)
The sequence approaches **3** (A1)

12.5 Show that $x^2 - 3x - 1 = 0$ can be rearranged to give $x = 3 + 1/x$. Use $x_{n+1} = 3 + 1/x_n$ with $x_0 = 3$ to find the positive solution to 2 decimal places. **5 marks** | Grade 7

$x^2 = 3x + 1$ (M1)
Divide by x: $x = 3 + 1/x$ (A1)
$x_1 = 3.3333$, $x_2 = 3.3$, $x_3 = 3.3030$ (M1 for at least two correct values)
$x_4 = 3.3028$, $x_5 = 3.3028$ (A1)
3.30 (A1)

12.6 The diagram shows the graphs of $y = x$ and $y = \sqrt[3]{(3x + 5)}$. The iteration $x_{n+1} = \sqrt[3]{(3x_n + 5)}$ starts at $x_0 = 1$. **4 marks** | Grade 7

(a) Write down the value of x_1. *[1]*
$\sqrt[3]{8} =$ **2** (B1)

(b) Does the iteration converge or diverge? Use the diagram to explain. *[2]*
Converges (B1)
The steps get smaller and move in towards the point where the curve meets $y = x$ (B1)

(c) Estimate the solution of $x^3 - 3x - 5 = 0$ from the diagram. *[1]*
2.3 (accept 2.2 to 2.4) (B1)

12.7 $x^3 - 3x - 5 = 0$ has a root between 2 and 3. Use a decimal search to find the root to 1 decimal place. Show your values of f(x). **4 marks** | Grade 8

f(2.2) = −0.952 and f(2.3) = 0.267 (M1)
Root between 2.2 and 2.3 (A1)
f(2.25) = −0.359, negative, so root between 2.25 and 2.3 (M1)
x = 2.3 (A1)

12.8 Show that $x^3 + 4x - 1 = 0$ has a root between 0 and 1. Rearrange the equation to give $x = 1/(x^2 + 4)$. Use $x_{n+1} = 1/(x_n^2 + 4)$ with $x_0 = 0.2$ to find the root to 3 decimal places. **5 marks** | Grade 8

f(0) = −1, f(1) = 4, change of sign (B1)
$x(x^2 + 4) = 1$ so $x = 1/(x^2 + 4)$ (M1)
$x_1 = 0.2475$, $x_2 = 0.2462$, $x_3 = 0.2463$ (M1)
Values settling at 0.24627 (M1)
0.246 (A1)

12.9 $x_{n+1} = \sqrt{(x_n + 6)}$. The sequence $x_0, x_1, x_2, \ldots$ converges to a limit L. Find the exact value of L. **4 marks** | Grade 9

$L = \sqrt{(L + 6)}$ (M1)
$L^2 = L + 6$ so $L^2 - L - 6 = 0$ (M1)
$(L - 3)(L + 2) = 0$ (A1)
L cannot be negative as all terms are square roots, so **L = 3** (A1)

12.10 Show that the iteration $x_{n+1} = 1/(x_n + 1)$ has a limit L satisfying $L^2 + L - 1 = 0$, and find L to 3 decimal places. **5 marks** | Grade 9

$L = 1/(L + 1)$ (M1)
$L(L + 1) = 1$ so $L^2 + L - 1 = 0$ (A1)
$L = [-1 + \sqrt{5}]/2$ (the positive root, since all terms are positive) (M1 A1)
L = 0.618 (A1)

Error log

Use this table to record where marks are lost. Patterns tell you what to practise.

Question	Marks lost	Knowledge	Method	Accuracy	Presentation	Action