GCSE MATHEMATICS | HIGHER TIER | YEAR 10

Number Lesson Workbook

Fractions, decimals and percentages, percentage change, reverse percentages, standard form, indices, surds and bounds

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Written for Edexcel (1MA1) and AQA (8300) Higher tier

Inside this workbook

1. Fractions, decimals and percentages
2. Percentage change
3. Reverse percentages
4. Standard form
5. Indices (Higher)
6. Surds (Higher)
7. Upper and lower bounds (Higher)

Revision sheet
15 classwork questions
30 homework questions
Full worked answers and mark schemes

Student name: _____

Class / date started: _____

Target grade: _____

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How to use this workbook

1. **Read** the key ideas for the topic. Learn the tables and rules.
2. **Cover the solution** to each worked example and try it yourself, then compare.
3. **Attempt** the exam-style questions under timed conditions, then mark them with the scheme.
4. **Classwork** is done in the lesson with your tutor. **Homework** is done independently, without notes at first.
5. **Mark your own work** using the answer section, record your score in the tracker, and tick topics to revisit.

> **About the exam-style questions.** All questions in this workbook are original. They are written to match the style, command words, structure and mark allocation of Edexcel GCSE (9-1) Mathematics 1MA1 and AQA GCSE (9-1) Mathematics 8300 Higher papers from 2018 to 2026. They are not copied from past papers. Real past papers and mark schemes are free on the Pearson Edexcel and AQA websites.

Mark scheme codes

Code	Meaning
M1	Method mark: a correct method is shown, even if there is an arithmetic slip.
A1	Accuracy mark: a correct answer, usually only awarded if the method mark was earned.
B1	Independent mark: awarded for a correct statement or answer without needing a method.

Command words

Command	What the examiner wants
Work out	Calculate and show your working. A correct answer alone may not earn full marks.
Show that	

Command	What the examiner wants
	Reach the given answer with clear, complete steps. You cannot work backwards from the answer.
Write down	No working required. Answer directly.
Simplify / Expand / Solve	Give the answer in its simplest form, multiply out brackets, or find the value of the unknown.
Give a reason	Write a short sentence linking your numbers to your conclusion.

Calculator guidance: papers 1H are non-calculator on both boards, so fractions, surds, indices and standard form must be done by hand. Papers 2H and 3H are calculator papers, where percentage change, compound interest and bounds calculations are common.

Topic 1: Fractions, Decimals and Percentages

These three are different ways of writing the same number. Fluency here supports almost every other Number topic, and it is tested on both the non-calculator and calculator papers.

Key ideas

Equivalence you must know

Fraction	Decimal	Percentage
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{1}{10}$	0.1	10%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{1}{3}$	0.3	33.3%
$\frac{2}{3}$	0.6	66.6%
$\frac{1}{20}$	0.05	5%

Converting between forms

- Fraction to decimal: divide the numerator by the denominator ($\frac{3}{8} = 3 \div 8 = 0.375$).
- Decimal to percentage: multiply by 100 (0.375 = 37.5%).
- Percentage to fraction: write over 100 and simplify (35% = $\frac{35}{100} = \frac{7}{20}$).

Adding and subtracting fractions

- Find a common denominator (the lowest common multiple of the denominators), then add or subtract the numerators only.
- For mixed numbers, convert to improper fractions first: $3\frac{2}{5} = \frac{17}{5}$.
- Always simplify the final answer and, if asked, write it as a mixed number.

Multiplying and dividing fractions

- Multiply: multiply the numerators and multiply the denominators. Cancel common factors first to keep numbers small.
- Divide: Keep the first fraction, Flip the second, then multiply (KFC).
- Never find a common denominator for multiplication or division.

Fractions and percentages of amounts

- Fraction of an amount: divide by the denominator, multiply by the numerator.
- One quantity as a fraction of another: write quantity over total, then simplify.
- Multi-step 'of the remainder' questions: work out what is left after each step, or multiply the fractions together.

Terminating and recurring decimals

- A fraction in its simplest form is a terminating decimal only if the denominator has no prime factors except 2 and 5. So $\frac{7}{40}$ terminates ($40 = 2^3 \times 5$) but $\frac{5}{12}$ recurs ($12 = 2^2 \times 3$).
- Recurring decimals are written with a dot over the digit that repeats ($0.\dot{3}$) or over the first and last digit of a repeating block ($0.\dot{2}\dot{7}$).
- To convert recurring to a fraction: let x equal the decimal, multiply by 10, 100 or 1000 so that the recurring parts line up, then subtract.

Worked examples

Example 1.1: Subtracting mixed numbers

Work out $3\frac{2}{5} - 1\frac{3}{4}$, giving your answer as a mixed number.

Convert: $3\frac{2}{5} = \frac{17}{5}$ and $1\frac{3}{4} = \frac{7}{4}$.

Common denominator is 20: $\frac{68}{20} - \frac{35}{20} = \frac{33}{20}$.

Convert back: $\frac{33}{20} = 1\frac{13}{20}$.

Answer: $1\frac{13}{20}$

Example 1.2: Dividing mixed numbers

Work out $2\frac{2}{3} \div 1\frac{1}{9}$.

Improper fractions: $\frac{8}{3} \div \frac{10}{9}$.

Keep, Flip, multiply: $\frac{8}{3} \times \frac{9}{10} = \frac{72}{30}$ (or cancel the 3 and 9 first to get $\frac{8}{1} \times \frac{3}{10}$).

Simplify: $\frac{72}{30} = \frac{12}{5} = 2\frac{2}{5}$.

Answer: $2\frac{2}{5}$

Example 1.3: Fraction, decimal and percentage

Write $\frac{7}{40}$ as a decimal and as a percentage.

$7 \div 40 = 0.175$.

$0.175 \times 100 = 17.5\%$.

Answer: 0.175 = 17.5%

Example 1.4: Recurring decimal to fraction (pure recurring)

Show that $0.\dot{5}\dot{4} = \frac{6}{11}$.

Let x = 0.545454... Two digits repeat, so multiply by 100: 100x = 54.545454...

Subtract: 100x − x = 54.5454... − 0.5454... so 99x = 54.

$x = \frac{54}{99} = \frac{6}{11}$ (divide top and bottom by 9).

Answer: $\frac{6}{11}$

Example 1.5: Recurring decimal to fraction (mixed recurring)

Convert $0.2\dot{1}$ (that is 0.2111...) to a fraction.

Let x = 0.2111... Then 10x = 2.111... and 100x = 21.111...

Subtract 10x from 100x: 90x = 19.

$x = \frac{19}{90}$.

Answer: $\frac{19}{90}$

Common mistakes to avoid

- Adding the denominators as well as the numerators ($\frac{1}{2} + \frac{1}{3}$ is not $\frac{2}{5}$).
- Finding a common denominator when multiplying or dividing fractions.
- Flipping the wrong fraction when dividing. Only the second fraction flips.
- Writing $0.\dot{3}$ as 0.3 or 0.33 in a 'show that' question. The dot notation matters.
- Leaving an answer unsimplified, which can lose the final accuracy mark.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 1.1 (Edexcel-style) Non-calculator | 3 marks

Show that $3\frac{1}{2} \div 2\frac{1}{3} = 1\frac{1}{2}$.

Mark scheme and worked solution

$\frac{7}{2} \div \frac{7}{3}$ (M1 for both converted to improper fractions)

$= \frac{7}{2} \times \frac{3}{7}$ (M1 for flipping the second fraction and multiplying)

$= \frac{21}{14} = \frac{3}{2} = 1\frac{1}{2}$ (A1 for correct simplification to the given answer)

Exam-style 1.2 (AQA-style) Non-calculator | 3 marks

Tom has £60. He spends $\frac{2}{5}$ of it on a book. He spends $\frac{1}{4}$ of the money that is left on lunch. How much money does he have left now?

Mark scheme and worked solution

$\frac{2}{5}$ of 60 = £24, so £36 is left (M1)

$\frac{1}{4}$ of 36 = £9 (M1)

36 − 9 = £27 (A1)

Topic 2: Percentage Change

Percentage change questions are worth marks every series, often in real-life contexts: sales, interest, population and depreciation. The multiplier method is the quickest and most reliable approach and it is the only one you need.

Key ideas

The multiplier method

- Increase by r%: multiply by $1 + \frac{r}{100}$.
- Decrease by r%: multiply by $1 - \frac{r}{100}$.
- Examples: +5% is × 1.05, −5% is × 0.95, +12.5% is × 1.125, −30% is × 0.7, +120% is × 2.2.

Percentage change formula

- Percentage change = $\frac{\text{actual change}}{\text{original value}} \times 100$.
- 'Original' is always the starting value, the one you are comparing against, not the final value.
- Percentage profit and percentage loss work in exactly the same way.

Repeated percentage change

- Multiply the multipliers together. Percentages cannot be added or subtracted across different years because each one acts on a different amount.
- Compound interest: $A = P \times (1 + \frac{r}{100})^n$, where P is the amount invested, r the yearly rate and n the number of years.
- Depreciation uses the same idea with a multiplier below 1.

Simple interest (for comparison)

- Simple interest is the same amount every year: Interest = $\frac{P \times r \times n}{100}$.
- Compound interest grows faster because interest is earned on previous interest.

Worked examples

Example 2.1: Percentage increase

Increase £64 by 35%.

Multiplier = 1.35.

64 × 1.35 = 86.4.

Answer: £86.40

Example 2.2: Percentage decrease

A bag of flour weighs 250 g. After baking, 18% of it is gone. How much is left?

Multiplier = $1 - 0.18 = 0.82$.

$250 \times 0.82 = 205$.

Answer: 205 g

Example 2.3: Finding the percentage change

Maya's hourly pay rises from £45 to £58.50. Find the percentage increase.

Change = $58.50 - 45 = 13.50$.

Percentage change = $\frac{13.5}{45} \times 100 = 30\%$.

Answer: 30% increase

Example 2.4: Successive changes

A price rises by 10% and then falls by 10%. Is it back to the start?

Multipliers: $1.1 \times 0.9 = 0.99$.

0.99 means 99% of the original, a 1% decrease overall. The second 10% is taken from a bigger number.

Answer: 1% decrease overall

Example 2.5: Compound interest

£8000 is invested for 5 years at 2.5% compound interest per year. Find the interest earned.

Multiplier per year = 1.025.

Final amount = $8000 \times 1.025^5 = £9051.27$ (to the nearest penny).

Interest = $9051.27 - 8000 = £1051.27$.

Answer: £1051.27

Example 2.6: Depreciation

A car worth £18 000 loses 15% of its value each year. Find its value after 3 years.

Multiplier = 0.85, so value = $18\,000 \times 0.85^3$.

$0.85^3 = 0.614125$, so value = £11 054.25.

Answer: £11 054.25

Common mistakes to avoid

- Calculating percentage change as a fraction of the new value instead of the original.
- Adding percentages across years (10% up then 10% down is not 0%).
- Using the compound interest formula for simple interest questions, and the reverse.
- Rounding the multiplier or the first year's value too early. Keep full calculator values and round at the end.
- Giving 'increase' or 'decrease' without saying which, when the question asks for a percentage change.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 2.1 (Edexcel-style) Calculator | 3 marks

Rafa invests £6000 in an account that pays 4% compound interest per year. Work out how much interest he has earned after 3 years.

Mark scheme and worked solution

Multiplier 1.04, so 6000×1.04^3 (M1)
$= 6749.184$ which is £6749.18 (A1)
Interest $= 6749.18 - 6000 =$ £749.18 (A1)

Exam-style 2.2 (AQA-style) Calculator | 4 marks

Sam's weekly pay rises from £320 to £352. Alex's weekly pay rises from £410 to £447.50. Whose pay had the greater percentage increase? You must show your working.

Mark scheme and worked solution

Sam: change = 32, $\frac{32}{320} \times 100 = 10\%$ (M1 for method, A1 for 10%)

Alex: change = 37.50, $\frac{37.5}{410} \times 100 = 9.15\%$ (3 s.f.) (M1, A1)

Sam had the greater percentage increase. (Conclusion supported by both figures.)

Topic 3: Reverse Percentages

In a reverse percentage question you are given the value after a percentage change and asked for the original. This is a classic Higher-tier trap: the wrong method looks so natural that many students use it.

Key ideas

The core idea

- Original × multiplier = new value.
- So: Original = new value ÷ multiplier.
- Never take the percentage off the new value. The percentage was calculated from the original amount, not the new one.

Steps to follow

- 1. Decide whether the change was an increase or a decrease, and write the multiplier.
- 2. Divide the given (final) amount by that multiplier.
- 3. Check by applying the multiplier forwards to your answer. You should get the number in the question.

Where you will meet it

- VAT and tax: price including 20% VAT = original × 1.2.
- Sale prices, discounts and price rises.
- Repeated changes: multiply the multipliers first, then divide by the combined multiplier.

Worked examples

Example 3.1: Reverse after an increase

After a 15% increase, a price is £69. What was the original price?

Multiplier = 1.15, so original = 69 ÷ 1.15.

69 ÷ 1.15 = 60. Check: 60 × 1.15 = 69. ✓

Answer: £60

Example 3.2: Reverse after a decrease

A jumper costs £52 in a sale after a 35% reduction. Find the normal price.

Multiplier = 1 − 0.35 = 0.65.

52 ÷ 0.65 = 80. Check: 80 × 0.65 = 52. ✓

Answer: £80

Example 3.3: The wrong method, and why it fails

A meal costs £90 including 20% VAT. Find the price without VAT.

Wrong: 20% of 90 = 18, and 90 − 18 = 72. (The 20% is of the original price, not of £90.)

Right: the original is multiplied by 1.2, so original = 90 ÷ 1.2 = 75. Check: 75 × 1.2 = 90. ✓

Answer: £75

Example 3.4: Reverse with two changes

A painting's value rose by 10% and then fell by 20% to £792. What was its value before the changes?

Combined multiplier = 1.1 × 0.8 = 0.88.

792 ÷ 0.88 = 900. Check: 900 × 1.1 × 0.8 = 792. ✓

Answer: £900

Common mistakes to avoid

- Finding the percentage of the final amount and adding or subtracting it (the wrong method above).
- Using the multiplier for the wrong direction, for example dividing by 0.65 when the item went up.
- Forgetting to combine multipliers for two-stage problems and reversing each stage badly.
- Not checking the answer forwards. A 10-second check catches nearly every error.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 3.1 (Edexcel-style) — Calculator | 3 marks

In a sale, all normal prices are reduced by 30%. The sale price of a sofa is £455. Work out the normal price of the sofa.

Mark scheme and worked solution

100% − 30% = 70%, multiplier 0.7 (M1)

455 ÷ 0.7 (M1)

£650 (A1)

Exam-style 3.2 (AQA-style) — Calculator | 4 marks

The price of a house increased by 8% in 2022 and then by 5% in 2023. At the end of 2023 it was worth £238 140. Work out its price at the start of 2022.

Mark scheme and worked solution

Multipliers 1.08 and 1.05 (B1)

1.08 × 1.05 = 1.134 (M1)

238 140 ÷ 1.134 (M1)

£210 000 (A1)

Topic 4: Standard Form

Standard form is the way scientists write very large and very small numbers. Exam questions test the definition, conversions and calculations, often in a science or real-world context.

Key ideas

Definition

- A number in standard form is written $a \times 10^n$, where $1 \leq a < 10$ and n is an integer (positive, negative or zero).
- Large numbers have a positive power: $56\,000\,000 = 5.6 \times 10^7$.
- Small numbers have a negative power: $0.000\,0306 = 3.06 \times 10^{-5}$. The negative power does not make the number negative.

Converting

- Count how many places the decimal point moves to leave a number between 1 and 10. That count is the power.
- Moving left gives a positive power, moving right gives a negative power.
- Check: 25×10^3 is not in standard form because 25 is not between 1 and 10. It is 2.5×10^4.

Calculating without a calculator

- Multiply: multiply the front numbers and add the powers of 10.
- Divide: divide the front numbers and subtract the powers of 10.
- Add or subtract: first write both numbers with the same power of 10, then add or subtract the front numbers.
- Finish by checking the front number is between 1 and 10 and adjusting if it is not.

Calculator tip

- Use the $\times 10^x$ or EXP key. Enter 3.2×10^5 as 3.2, EXP, 5. Do not type 3.2 × 10 ^ 5 unless you bracket carefully.
- Write the calculator display in full standard form. Never write 3.2E5 or similar in an answer.

Worked examples

Example 4.1: Writing in standard form

Write (a) 56 000 000 and (b) 0.000 0306 in standard form.

(a) The point moves 7 places left: 5.6×10^7.

(b) The point moves 5 places right: 3.06×10^{-5}.

Answer: (a) 5.6×10^7 (b) 3.06×10^{-5}

Example 4.2: Writing as an ordinary number

Write 4.7×10^{-3} as an ordinary number.

A power of -3 means move the point 3 places to the right.

$4.7 \times 10^{-3} = 0.0047$.

Answer: 0.0047

Example 4.3: Multiplying

Work out $(6.4 \times 10^{5}) \times (2.5 \times 10^{-2})$, giving your answer in standard form.

Front numbers: $6.4 \times 2.5 = 16$. Powers: $10^{5} \times 10^{-2} = 10^{3}$.

That gives 16×10^{3}. Since 16 is not between 1 and 10, rewrite: 1.6×10^{4}.

Answer: 1.6×10^{4}

Example 4.4: Dividing

Work out $(9.6 \times 10^{7}) \div (3.2 \times 10^{-2})$.

Front numbers: $9.6 \div 3.2 = 3$. Powers: $7 - (-2) = 9$.

Answer 3×10^{9}.

Answer: 3×10^{9}

Example 4.5: Adding

Work out $3.4 \times 10^{6} + 5.2 \times 10^{5}$.

Make the powers match: $5.2 \times 10^{5} = 0.52 \times 10^{6}$.

$3.4 \times 10^{6} + 0.52 \times 10^{6} = 3.92 \times 10^{6}$.

Answer: 3.92×10^{6}

Example 4.6: In context

A virus is 1.2×10^{-7} m long. How many could be placed end to end along 1 mm (10^{-3} m)? Give your answer to 3 s.f.

Divide the total length by one length: $\dfrac{10^{-3}}{1.2 \times 10^{-7}}$.

$= 0.8333... \times 10^{4} = 8.33 \times 10^{3}$ (3 s.f.).

Answer: 8.33×10^{3}

Common mistakes to avoid

- Leaving the front number outside the range 1 to 10 (for example 16×10^{3}).
- Thinking a negative power gives a negative number.
- Adding powers when adding numbers ($3 \times 10^{4} + 2 \times 10^{4}$ is 5×10^{4}, not 5×10^{8}).
- Counting decimal places incorrectly. Count the places the point moves, not the zeros.
- Writing calculator notation (3.2E5) as the final answer.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 4.1 (Edexcel-style) — Calculator | 2 marks

A bacterium has mass 2.5×10^{-12} g. A culture contains 4×10^{9} bacteria. Work out the total mass of the bacteria. Give your answer in standard form.

Mark scheme and worked solution

$2.5 \times 4 = 10$ and $10^{-12} \times 10^{9} = 10^{-3}$, giving 10×10^{-3} (M1)

1×10^{-2} g (A1) (must be converted into standard form)

Exam-style 4.2 (AQA-style) — Non-calculator | 3 marks

Work out $2.4 \times 10^{8} + 6.5 \times 10^{7}$. Give your answer in standard form.

Mark scheme and worked solution

Write both with the same power: $24 \times 10^{7} + 6.5 \times 10^{7}$ (M1)

$= 30.5 \times 10^{7}$ (A1)

$= 3.05 \times 10^{8}$ (A1)

Topic 5: Indices (Higher)

Higher tier indices go beyond whole-number powers to negative and fractional indices, and to solving equations where the unknown is in the power. Learn the laws and the roots and powers table; both are heavily used.

Key ideas

The laws of indices

Law	Rule	Example
Multiply	$a^m \times a^n = a^{m+n}$	$x^3 \times x^4 = x^7$
Divide	$a^m \div a^n = a^{m-n}$	$x^8 \div x^5 = x^3$
Power of a power	$(a^m)^n = a^{mn}$	$(x^2)^5 = x^{10}$
Power of a product	$(ab)^n = a^n b^n$	$(3x)^2 = 9x^2$
Zero index	$a^0 = 1$	$7^0 = 1$

Negative and fractional indices

Rule	Meaning	Example
Negative index	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8}$
Reciprocal of a fraction	$(\frac{a}{b})^{-n} = (\frac{b}{a})^n$	$(\frac{2}{3})^{-2} = \frac{9}{4}$
Fractional index (root)	$a^{\frac{1}{n}}$ = the n-th root of a	$25^{\frac{1}{2}} = 5$, $27^{\frac{1}{3}} = 3$
General fractional index	$a^{\frac{m}{n}}$ = (n-th root of a)^m	$8^{\frac{2}{3}} = 2^2 = 4$

Order of working for fractional and negative powers

- 1. Deal with the negative sign first by taking the reciprocal.
- 2. Take the root (the denominator of the index).
- 3. Then apply the power (the numerator of the index). Taking the root first keeps the numbers small.

Solving index equations

- Write both sides as a power of the same base, then equate the powers.
- Example: $4^x = 2^5$. Write 4 as 2^2, so $2^{2x} = 2^5$ and x = 2.5.
- Values worth learning: squares to 15^2, cubes 1, 8, 27, 64, 125, 1000, and powers of 2 up to $2^{10} = 1024$.

Worked examples

Example 5.1: Simplifying algebraic powers

Simplify $3x^4y \times 5x^2y^3$.

Multiply the numbers: $3 \times 5 = 15$.

Add powers of x: $x^{4+2} = x^6$. Add powers of y: $y^{1+3} = y^4$.

Answer: $15x^6y^4$

Example 5.2: Fractional index

Work out $125^{\frac{2}{3}}$.

Cube root first: $125^{\frac{1}{3}} = 5$.

Then square: $5^2 = 25$.

Answer: 25

Example 5.3: Negative fractional index

Work out $\left(\frac{1}{4}\right)^{-\frac{3}{2}}$.

The negative index flips the fraction: $\left(\frac{1}{4}\right)^{-\frac{3}{2}} = 4^{\frac{3}{2}}$.

Square root first: $4^{\frac{1}{2}} = 2$. Then cube: $2^3 = 8$.

Answer: 8

Example 5.4: Fractional index with algebra

Simplify $(27a^6 \div b^{-3})^{\frac{1}{3}}$.

Apply the power to each part: $27^{\frac{1}{3}} = 3$, $(a^6)^{\frac{1}{3}} = a^2$, $(b^{-3})^{\frac{1}{3}} = b^{-1}$.

Dividing by b^{-1} is multiplying by b. So the result is $3a^2b$.

Answer: $3a^2b$

Example 5.5: Solving: single base

Solve $2^x = \frac{1}{32}$.

$\frac{1}{32} = 2^{-5}$, so $2^x = 2^{-5}$.

$x = -5$.

Answer: $x = -5$

Example 5.6: Solving: different bases

Solve $4^{x+1} = 8^x$.

Write both with base 2: $(2^2)^{x+1} = (2^3)^x$, so $2^{2x+2} = 2^{3x}$.

Equate powers: $2x + 2 = 3x$, so $x = 2$.

Answer: $x = 2$

Common mistakes to avoid

- Multiplying the powers when they should be added ($x^3 \times x^4$ is x^7, not x^{12}).
- Treating a negative index as a negative number (2^{-3} is $\frac{1}{8}$, not -8).
- Applying a power to only part of a bracket: $(3x)^2 = 9x^2$, not $3x^2$.
- Raising to the power before taking the root, giving huge numbers that are hard to handle.
- Forgetting that $5^0 = 1$, not 0.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 5.1 (Edexcel-style) — Non-calculator | 3 marks

Simplify fully $(3x^2y^3)^3 \div (9xy^4)$.

Mark scheme and worked solution

$(3x^2y^3)^3 = 27x^6y^9$ (M1)

$27 \div 9 = 3$, $x^6 \div x = x^5$, $y^9 \div y^4 = y^5$ (M1)

$3x^5y^5$ (A1)

Exam-style 5.2 (AQA-style) — Non-calculator | 3 marks

Solve $2^{3x+1} \div 2^{x-3} = 64$.

Mark scheme and worked solution

Subtract the powers: $2^{(3x+1)-(x-3)} = 2^{2x+4}$ (M1)

$64 = 2^6$, so $2x + 4 = 6$ (M1)

$x = 1$ (A1)

Topic 6: Surds (Higher)

A surd is a root that cannot be written as a whole number or a fraction, such as $\sqrt{2}$ or $\sqrt{7}$. Surds let you give exact answers instead of rounded decimals. Exam questions ask you to simplify, expand and rationalise.

Key ideas

What is a surd?

- $\sqrt{4} = 2$ and $\sqrt{\frac{9}{16}} = \frac{3}{4}$ are rational, so they are not surds. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{12}$ are irrational surds.
- Surds give exact values, so leave your answer in surd form unless told to use a decimal.
- Know the square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144.

Rules

Rule	Example
$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$	$\sqrt{2} \times \sqrt{8} = \sqrt{16} = 4$
$\sqrt{a} \div \sqrt{b} = \sqrt{(a \div b)}$	$\sqrt{50} \div \sqrt{2} = \sqrt{25} = 5$
$\sqrt{a} \times \sqrt{a} = a$	$\sqrt{7} \times \sqrt{7} = 7$
$\sqrt{(a + b)}$ is not $\sqrt{a} + \sqrt{b}$	$\sqrt{(9 + 16)} = 5$, but $\sqrt{9} + \sqrt{16} = 7$

Simplifying a surd

- Find the largest square number that divides into the number under the root.
- $\sqrt{72} = \sqrt{(36 \times 2)} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$.
- You can only add or subtract 'like' surds, which have the same surd part: $3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$.

Expanding brackets

- Multiply every term in the first bracket by every term in the second (FOIL or a grid).
- Difference of two squares: $(a + b)(a - b) = a^2 - b^2$. With surds, $(3 + \sqrt{5})(3 - \sqrt{5}) = 9 - 5 = 4$, which is rational.

Rationalising the denominator

- To remove a surd from the bottom, multiply the top and bottom by a suitable surd.
- Single surd: $\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}}{b}$.
- Two-term denominator, such as $b + \sqrt{c}$: multiply by the conjugate $b - \sqrt{c}$. The denominator becomes $b^2 - c$, which is rational.

Worked examples

Example 6.1: Simplifying

Write $\sqrt{72}$ in the form $a\sqrt{b}$ where a and b are integers and b is as small as possible.

$\sqrt{72} = \sqrt{(36 \times 2)} = \sqrt{36} \times \sqrt{2}$.

$= 6\sqrt{2}$.

Answer: $6\sqrt{2}$

Example 6.2: Adding and subtracting surds

Simplify $3\sqrt{8} + \sqrt{50} - \sqrt{18}$.

$3\sqrt{8} = 3 \times 2\sqrt{2} = 6\sqrt{2}$; $\sqrt{50} = 5\sqrt{2}$; $\sqrt{18} = 3\sqrt{2}$.

$6\sqrt{2} + 5\sqrt{2} - 3\sqrt{2} = 8\sqrt{2}$.

Answer: $8\sqrt{2}$

Example 6.3: Expanding

Expand and simplify $(2 + \sqrt{3})(4 - \sqrt{3})$.

$2 \times 4 = 8$; $2 \times (-\sqrt{3}) = -2\sqrt{3}$; $\sqrt{3} \times 4 = 4\sqrt{3}$; $\sqrt{3} \times (-\sqrt{3}) = -3$.

$8 - 3 + (-2\sqrt{3} + 4\sqrt{3}) = 5 + 2\sqrt{3}$.

Answer: $5 + 2\sqrt{3}$

Example 6.4: Rationalising a single surd

Rationalise the denominator of $\dfrac{14}{\sqrt{7}}$.

Multiply top and bottom by $\sqrt{7}$: $\dfrac{14\sqrt{7}}{7}$.

$= 2\sqrt{7}$.

Answer: $2\sqrt{7}$

Example 6.5: Rationalising a two-term denominator

Write $\dfrac{6}{4 - \sqrt{10}}$ in the form $a + b\sqrt{c}$.

Multiply top and bottom by $(4 + \sqrt{10})$: $\dfrac{6(4 + \sqrt{10})}{(4 - \sqrt{10})(4 + \sqrt{10})}$.

Denominator: $16 - 10 = 6$. So the result is $\dfrac{6(4 + \sqrt{10})}{6} = 4 + \sqrt{10}$.

Answer: $4 + \sqrt{10}$

Example 6.6: Exact lengths

A right-angled triangle has shorter sides $2\sqrt{3}$ cm and 4 cm. Find the exact length of the hypotenuse.

Pythagoras: $h^2 = (2\sqrt{3})^2 + 4^2 = 12 + 16 = 28$.

$h = \sqrt{28} = \sqrt{(4 \times 7)} = 2\sqrt{7}$ cm.

Answer: $2\sqrt{7}$ cm

Common mistakes to avoid

- Writing $\sqrt{(a + b)} = \sqrt{a} + \sqrt{b}$, or $\sqrt{(a - b)} = \sqrt{a} - \sqrt{b}$.
- Stopping at $\sqrt{72}$ or $3\sqrt{8}$ without simplifying fully.
- Forgetting to multiply the numerator as well as the denominator when rationalising.
- In $(a + \sqrt{b})^2$, writing $a^2 + b$ and missing the middle term $2a\sqrt{b}$.
- Adding unlike surds: $2\sqrt{3} + 5\sqrt{2}$ cannot be combined.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 6.1 (Edexcel-style) — Non-calculator | 3 marks

Show that $\dfrac{\sqrt{18} + 6}{\sqrt{2}}$ can be written as $3 + 3\sqrt{2}$.

Mark scheme and worked solution

Split the fraction: $\dfrac{\sqrt{18}}{\sqrt{2}} + \dfrac{6}{\sqrt{2}}$ (M1)

$\dfrac{\sqrt{18}}{\sqrt{2}} = \sqrt{9} = 3$ and $\dfrac{6}{\sqrt{2}} = \dfrac{6\sqrt{2}}{2} = 3\sqrt{2}$ (M1)

$3 + 3\sqrt{2}$ (A1)

Exam-style 6.2 (AQA-style) — Non-calculator | 4 marks

The area of a rectangle is $(8 + 5\sqrt{3})$ cm². Its length is $(2 + \sqrt{3})$ cm. Work out the width. Give your answer in the form $a + b\sqrt{3}$.

Mark scheme and worked solution

Width $= \dfrac{8 + 5\sqrt{3}}{2 + \sqrt{3}}$ (M1)

Multiply top and bottom by $(2 - \sqrt{3})$ (M1)

Numerator: $16 - 8\sqrt{3} + 10\sqrt{3} - 15 = 1 + 2\sqrt{3}$. Denominator: $4 - 3 = 1$ (M1)

Width $= (1 + 2\sqrt{3})$ cm (A1)

Topic 7: Upper and Lower Bounds (Higher)

Measurements and rounded values are never exact, so any answer calculated from them lies in a range. Bounds questions are a staple of the Higher papers and often carry 4 to 6 marks.

Key ideas

Bounds from rounding

- If a value is rounded to the nearest unit u, the true value is within half a unit either side.
- Lower bound = value $-\frac{u}{2}$. Upper bound = value $+\frac{u}{2}$.
- Error interval: lower bound $\leq x <$ upper bound. The upper bound itself is not included, but you still use it in calculations.

Reading the accuracy

Value	Rounded to	Error interval
3.8	1 decimal place	$3.75 \leq x < 3.85$
250	2 significant figures	$245 \leq x < 255$
4500	nearest 100	$4450 \leq x < 4550$
12	nearest whole number	$11.5 \leq x < 12.5$

Calculations with bounds

Calculation	For the upper bound	For the lower bound
a + b	UB of a + UB of b	LB of a + LB of b
a − b	UB of a − LB of b	LB of a − UB of b
a × b	UB of a × UB of b	LB of a × LB of b
a ÷ b	UB of a ÷ LB of b	LB of a ÷ UB of b

Giving a final answer to a suitable accuracy

- Calculate both the upper and the lower bound of the answer.
- Round both to the same number of significant figures, increasing the accuracy until they differ. The last accuracy for which they match is the one you can justify.
- State the final answer to that accuracy and explain how you know.

Worked examples

Example 7.1: Error intervals

Write down the error interval for each value: (a) 3.8 to 1 d.p.; (b) 250 to 2 s.f.; (c) 4500 to the nearest 100.

(a) Half of 0.1 is 0.05: $3.75 \leq x < 3.85$.

(b) 2 s.f. means the nearest 10, half is 5: $245 \leq x < 255$.

(c) Half of 100 is 50: $4450 \leq x < 4550$.

Answer: (a) $3.75 \leq x < 3.85$ (b) $245 \leq x < 255$ (c) $4450 \leq x < 4550$

Example 7.2: Sum, difference and quotient

a = 7.2 and b = 3.5, both to 1 d.p. Bounds: $7.15 \leq a < 7.25$ and $3.45 \leq b < 3.55$.

UB of $a + b = 7.25 + 3.55 = 10.8$.

LB of $a - b = 7.15 - 3.55 = 3.6$ (lower a, upper b).

UB of $a \div b = 7.25 \div 3.45 = 2.10$ (3 s.f.) (upper a, lower b).

Answer: UB(a+b) = 10.8, LB(a−b) = 3.6, UB(a÷b) = 2.10

Example 7.3: Speed and a suitable degree of accuracy

A runner covers 64 m (to the nearest metre) in 3.2 s (to 1 d.p.). Find the speed to a suitable degree of accuracy.

Distance: $63.5 \leq d < 64.5$. Time: $3.15 \leq t < 3.25$.

Lower bound of speed = $63.5 \div 3.25 = 19.538...$ Upper bound = $64.5 \div 3.15 = 20.476...$

To 2 s.f. both are 20. To 3 s.f. they differ (19.5 and 20.5). So the speed is 20 m/s to 2 s.f.

Answer: 20 m/s (2 s.f.)

Common mistakes to avoid

- Using the nearest unit instead of half a unit (3.8 to 1 d.p. has bounds 3.75 and 3.85, not 3.7 and 3.9).
- Using the wrong combination for subtraction and division (you need the lower bound of the second number for an upper bound).
- Writing the error interval with ≤ on both sides. The upper end uses <.
- Saying the answer 'is 20.5' when both bounds do not round to 20.5. Check both before concluding.
- Rounding bounds before using them in the calculation.

Exam-style worked questions

Original questions written in the style of the board named. Try each one first, then read the mark scheme.

Exam-style 7.1 (Edexcel-style) Non-calculator | 2 marks

The length of a pencil is 14 cm, correct to the nearest centimetre. Write down the error interval for the length, l.

Mark scheme and worked solution

13.5 (B1)

$13.5 \leq l < 14.5$ (B1) (accept 14.49 recurring for the upper bound, but not 14.5 with ≤)

Exam-style 7.2 (AQA-style) Calculator | 3 marks

A block has mass 540 g, correct to the nearest 10 g, and volume 200 cm³, correct to the nearest 10 cm³. Work out the lower bound of the density. Give your answer to 3 significant figures.

Mark scheme and worked solution

LB of mass = 535 and UB of volume = 205 (B1)

Density = $\frac{\text{mass}}{\text{volume}}$, lower bound = 535 ÷ 205 (M1)

2.61 g/cm³ (3 s.f.) (A1)

Revision sheet: key facts

Keep this page in front of you for homework. Aim to recall it all from memory before the test.

Fractions, decimals, percentages

Add or subtract: common denominator. Multiply: top × top, bottom × bottom. Divide: Keep, Flip, multiply. Terminating decimal: denominator (simplest form) has only the factors 2 and 5. Recurring to fraction: multiply to line up the repeat, subtract.

Percentage change

Multiplier = $1 \pm \frac{r}{100}$. Change % = $\frac{\text{change}}{\text{original}} \times 100$. Compound: $A = P(1 + \frac{r}{100})^n$. Repeated change: multiply the multipliers.

Reverse percentages

Original = new ÷ multiplier. Never take the percentage off the new amount. Check by multiplying forwards.

Standard form

$a \times 10^n$ with $1 \leq a < 10$. Multiply: × the fronts, add the powers. Divide: ÷ the fronts, subtract the powers. Add or subtract: match the powers first.

Indices

$a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^0 = 1$; $a^{-n} = \frac{1}{a^n}$; $a^{\frac{m}{n}}$ = (n-th root of a)^m. Solve by writing both sides with the same base.

Surds

$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$; $\sqrt{a} \times \sqrt{a} = a$. Simplify using the largest square factor. Rationalise: $\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$; for $b + \sqrt{c}$ use the conjugate $b - \sqrt{c}$. $(a + b)(a - b) = a^2 - b^2$.

Bounds

Rounded to the nearest u: $\pm \frac{u}{2}$. Error interval: LB ≤ x < UB. Add: UB + UB. Subtract: UB − LB. Multiply: UB × UB. Divide: UB ÷ LB. Opposite combinations for the lower bound.

Classwork

15 questions, 50 marks. Work through them with your tutor. Show all working; method marks are awarded in the exam.

CW1 FDP — Non-calculator **[2 marks]**

Work out $\frac{2}{3} + \frac{3}{5}$. Give your answer as a mixed number.

(2)

Working

Answer: _____

CW2 FDP — Non-calculator **[4 marks]**

(a) Write $\frac{3}{8}$ as a decimal. **(1)**

(b) Show that $0.1\dot{3} = \frac{2}{15}$. ($0.1\dot{3}$ means 0.1333... with the 3 recurring.) **(3)**

Working

Answer: _____

CW3 Percentage change — Calculator **[2 marks]**

In a sale, the price of a phone is reduced by 15%. Before the sale the price was £480. Work out the sale price.

(2)

Working

Answer: _____

CW4 Percentage change — Calculator — **[4 marks]**

Lena puts £2500 into a savings account that pays 3% compound interest per year. Work out the total amount in the account after 4 years. Give your answer to the nearest penny.

(4)

Working

Answer: _____

CW5 Reverse % — Calculator — **[2 marks]**

The price of a ticket is increased by 20% to £54. Work out the price before the increase.

(2)

Working

Answer: _____

CW6 Reverse % — Calculator — **[4 marks]**

In a sale, all prices are reduced by 28%.

(a) The sale price of a coat is £68.40. Work out the normal price of the coat. **(3)**

(b) A scarf has a normal price of £15. Work out its sale price. **(1)**

Working

Answer: _____

CW7 Standard form — Non-calculator — **[2 marks]**

Write 0.0000472 in standard form.

(2)

Working

Answer: _____

CW8 Standard form — Calculator — **[4 marks]**

Work out the following. Give each answer in standard form.

(a) $(3.2 \times 10^5) \times (1.5 \times 10^{-3})$ **(2)**

(b) $(6 \times 10^8) \div (2.5 \times 10^3)$ **(2)**

Working

Answer: _____

CW9 Indices — Non-calculator — **[2 marks]**

Work out the value of $27^{\frac{2}{3}}$.

(2)

Working

Answer: _____

CW10 Indices — Non-calculator — **[4 marks]**

(a) Work out $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$. Give your answer as a fraction. **(2)**

(b) Solve $4^x = \frac{1}{8}$. **(2)**

Working

Answer: _____

CW11 Surds — Non-calculator — **[2 marks]**

Write $\sqrt{48}$ in the form $a\sqrt{3}$, where a is an integer.

(2)

Working

Answer: _____

CW12 Surds — Non-calculator — **[4 marks]**

(a) Rationalise the denominator of $\frac{10}{\sqrt{5}}$. Give your answer in its simplest form. **(2)**

(b) Expand and simplify $(3 + \sqrt{2})(5 - \sqrt{2})$. **(2)**

Working

Answer: _____

CW13 Surds — Non-calculator — **[6 marks]**

(a) Show that $\sqrt{75} + \sqrt{12} = 7\sqrt{3}$. **(2)**

(b) Rationalise the denominator of $\frac{6}{\sqrt{3} - 1}$ and simplify. Give your answer in the form $a + b\sqrt{3}$. **(2)**

(c) Expand and simplify $(3 + \sqrt{3})^2$. **(2)**

Working

Answer: _____

CW14 Bounds — Non-calculator — **[2 marks]**

The value of x is 6.4, correct to 1 decimal place. Write down the lower bound and the upper bound of x.

(2)

Working

Answer: _____

CW15 Bounds Calculator **[6 marks]**

A rectangle has length 12.5 cm and width 8.2 cm. Both measurements are correct to 1 decimal place.

(a) Work out the upper bound of the perimeter of the rectangle. **(2)**

(b) Work out the lower bound of the area of the rectangle. **(2)**

(c) Work out the upper bound of length ÷ width. Give your answer to 3 significant figures. **(2)**

Working

Answer: _____

Classwork score tracker

Topic	Marks available	My score	Need to revisit? (tick)
FDP	6		
Percentage change	6		
Reverse %	6		
Standard form	6		
Indices	6		
Surds	12		
Bounds	8		
Total	**50**		

Target: 70% or more is secure. Between 50% and 70%, revisit the worked examples. Below 50%, re-read the key ideas and repeat the topic with your tutor.

Homework

30 questions, 106 marks. Complete in sections of 10 over the week. Questions marked non-calculator must be done by hand.

HW1 FDP Non-calculator **[2 marks]**

Work out $3\frac{1}{4} \div 1\frac{5}{8}$.

(2)

Working

Answer: _____

HW2 FDP Calculator **[2 marks]**

Write these numbers in order of size. Start with the smallest. 0.7, $\frac{5}{8}$, 68%, $\frac{2}{3}$

(2)

Working

Answer: _____

HW3 FDP Non-calculator **[4 marks]**

Sam spends $\frac{1}{3}$ of his monthly pay on rent and $\frac{2}{5}$ of his monthly pay on food. He saves $\frac{3}{4}$ of the money that is left. Sam saves £84 each month. Work out his monthly pay.

(4)

Working

Answer: _____

HW4 FDP Non-calculator **[6 marks]**

(a) Show that $0.4\dot{5} = \frac{41}{90}$. ($0.4\dot{5}$ means 0.4555... with the 5 recurring.) **(3)**

(b) Work out $2\frac{2}{3} - 1\frac{4}{5}$. Give your answer as a fraction in its simplest form. **(3)**

Working

Answer: _____

HW5 Percentage change Calculator **[2 marks]**

Increase £36 by 12.5%.

(2)

Working

Answer: _____

HW6 Percentage change Calculator **[2 marks]**

The value of a car falls from £12 400 to £10 540. Work out the percentage decrease.

(2)

Working

Answer: _____

HW7 Percentage change Calculator **[4 marks]**

A house is valued at £240 000. In the first year its value increases by 5%. In the second year its value decreases by 3%.

(a) Work out the value of the house after two years. **(2)**

(b) Work out the overall percentage change over the two years. **(2)**

Working

Answer: _____

HW8 Percentage change Calculator **[4 marks]**

The population of a town is 18 000. It increases by 2.5% each year.

(a) Work out the population after 3 years. Give your answer to the nearest whole number. **(2)**

(b) In another town the population is multiplied by 1.04 each year. Work out the percentage increase over 2 years. **(2)**

Working

Answer: _____

HW9 Percentage change Calculator **[6 marks]**

Ava buys a car for £15 000. In the first year its value falls by 18%. In each later year, its value falls by 12% of its value at the start of that year.

(a) Work out the value of the car after 3 years. **(3)**

(b) After how many whole years will the car be worth less than £5000 for the first time? **(3)**

Working

Answer: _____

HW10 Reverse % Calculator **[2 marks]**

A television costs £330 including VAT at 20%. Work out the price of the television without VAT.

(2)

Working

Answer: _____

HW11 Reverse % Calculator **[4 marks]**

Jo's salary was increased by 4% to £31 200.

(a) Work out her salary before the increase. **(2)**

(b) A year later her salary of £31 200 is increased by a further 2.5%. Work out her new salary. **(2)**

Working

Answer: _____

HW12 Reverse % Calculator **[6 marks]**

Shop A sells a jacket for £84 after a 30% reduction. In Shop B the same jacket had a normal price £5 higher than the normal price in Shop A, and Shop B gives a 25% reduction.

(a) Work out the normal price of the jacket in Shop A. **(2)**

(b) Work out the sale price of the jacket in Shop B. **(2)**

(c) Which shop is cheaper and by how much? **(2)**

Working

Answer: _____

HW13 Standard form — Non-calculator **[2 marks]**

Write 3.6×10^{-4} as an ordinary number.

(2)

Working

Answer: _____

HW14 Standard form — Non-calculator **[2 marks]**

Work out $(4 \times 10^{6}) \times (7 \times 10^{-2})$. Give your answer in standard form.

(2)

Working

Answer: _____

HW15 Standard form — Calculator **[4 marks]**

The mass of the Earth is 5.97×10^{24} kg. The mass of the Moon is 7.35×10^{22} kg.

(a) How many times heavier is the Earth than the Moon? Give your answer to 3 significant figures. **(2)**

(b) Work out the total mass of the Earth and the Moon. Give your answer in standard form, correct to 3 significant figures. **(2)**

Working

Answer: _____

HW16 Standard form — Calculator — **[6 marks]**

Light travels at 3×10^8 metres per second. The distance from the Sun to Neptune is 4.5×10^9 kilometres.

(a) Work out the time, in seconds, for light to travel from the Sun to Neptune. Give your answer in standard form. **(4)**

(b) Write your answer to part (a) in hours and minutes. **(2)**

Working

Answer: _____

HW17 Indices — Non-calculator — **[2 marks]**

Simplify $(3x^4)^2 \times 2x^{-3}$.

(2)

Working

Answer: _____

HW18 Indices — Non-calculator — **[2 marks]**

Work out the value of $8^{-\frac{2}{3}}$. Give your answer as a fraction.

(2)

Working

Answer: _____

HW19 Indices — Non-calculator **[4 marks]**

(a) Solve $3^x = \frac{1}{81}$. **(2)**

(b) Solve $2^{3x-1} = 16$. **(2)**

Working

Answer: _____

HW20 Indices — Non-calculator **[4 marks]**

(a) Simplify fully $\left(\frac{16x^8}{9y^{-4}}\right)^{\frac{1}{2}}$. **(2)**

(b) Find the value of n when $9^n = 27^{n-1}$. **(2)**

Working

Answer: _____

HW21 Surds — Non-calculator **[2 marks]**

Simplify $\sqrt{98} - \sqrt{32}$. Give your answer in the form $a\sqrt{2}$.

(2)

Working

Answer: _____

HW22 Surds — Non-calculator **[2 marks]**

Expand and simplify $(\sqrt{5} + 2)(\sqrt{5} - 2)$.

(2)

Working

Answer: _____

HW23 Surds — Non-calculator **[4 marks]**

(a) Rationalise the denominator of $\frac{12}{3+\sqrt{5}}$. Give your answer in the form $a - b\sqrt{5}$. **(3)**

(b) Write $\sqrt{45}$ in the form $k\sqrt{5}$. **(1)**

Working

Answer: _____

HW24 Surds — Non-calculator **[4 marks]**

A right-angled triangle has two shorter sides of length $3\sqrt{2}$ cm and $\sqrt{14}$ cm. Work out the exact length of the hypotenuse. Give your answer in the form $a\sqrt{b}$ where a and b are integers.

(4)

Working

Answer: _____

HW25 Surds — Non-calculator **[6 marks]**

(a) Express $\frac{4+\sqrt{8}}{\sqrt{2}}$ in the form $a + b\sqrt{2}$. **(3)**

(b) Rationalise the denominator of $\frac{5+\sqrt{2}}{5-\sqrt{2}}$ and simplify. **(3)**

Working

Answer: _____

HW26 Bounds — Non-calculator **[2 marks]**

The mass of a parcel is 450 g, correct to the nearest 10 g. Write down the error interval for the mass, m.

(2)

Working

Answer: _____

HW27 Bounds — Non-calculator **[2 marks]**

a = 3.2 and b = 1.6, both correct to 1 decimal place. Work out the lower bound of a − b.

(2)

Working

Answer: _____

HW28 Bounds — Calculator **[4 marks]**

A car travels 240 m, correct to the nearest 10 m, in 8.0 seconds, correct to the nearest 0.1 second. Speed = distance ÷ time. Give your answers to 3 significant figures.

(a) Work out the upper bound of the speed of the car. **(2)**

(b) Work out the lower bound of the speed of the car. **(2)**

Working

Answer: _____

HW29 Bounds Calculator **[4 marks]**

A cylinder has radius 5 cm and height 12 cm. Both measurements are correct to the nearest centimetre. Work out the upper bound of the volume of the cylinder. Give your answer to the nearest whole number.

(4)

Working

Answer: _____

HW30 Bounds Calculator **[6 marks]**

A field is a rectangle. Its area is 3600 m², correct to the nearest 100 m². Its width is 45 m, correct to the nearest metre.

(a) Work out the lower bound of the length of the field. **(2)**

(b) Work out the upper bound of the length of the field. **(2)**

(c) Use your answers to find the length of the field to a suitable degree of accuracy. Give a reason for your answer. **(2)**

Working

Answer: _____

Homework score tracker

Topic	Marks available	My score	Need to revisit? (tick)
FDP	14		
Percentage change	18		
Reverse %	12		
Standard form	14		
Indices	12		
Surds	18		
Bounds	18		
Total	**106**		

Target: 70% or more is secure. Between 50% and 70%, revisit the worked examples. Below 50%, re-read the key ideas and repeat the topic with your tutor.

Answers and mark schemes

Classwork

CW1 (FDP, 2 marks)

$\frac{10}{15} + \frac{9}{15}$ (M1 common denominator)

$= \frac{19}{15} = 1\frac{4}{15}$ (A1)

CW2 (FDP, 4 marks)

(a) 0.375 (B1)

(b) x = 0.1333... so 10x = 1.333... and 100x = 13.333... (M1)

100x − 10x = 13.333... − 1.333... so 90x = 12 (M1)

$x = \frac{12}{90} = \frac{2}{15}$ (A1)

CW3 (Percentage change, 2 marks)

480 × 0.85 (M1)

£408 (A1)

CW4 (Percentage change, 4 marks)

Multiplier 1.03 (B1)

2500×1.03^4 (M1)

2813.772... (A1)

£2813.77 (A1)

CW5 (Reverse %, 2 marks)

54 ÷ 1.2 (M1)

£45 (A1)

CW6 (Reverse %, 4 marks)

(a) Multiplier 0.72 (B1) 68.40 ÷ 0.72 (M1) £95 (A1)

(b) 15 × 0.72 = £10.80 (B1)

CW7 (Standard form, 2 marks)

4.72×10^{-5} (B2). Award B1 for 4.72×10^{n} with n wrong, or for 47.2×10^{-6}.

CW8 (Standard form, 4 marks)

(a) $3.2 \times 1.5 = 4.8$ (M1), so 4.8×10^2 (A1)

(b) $6 \div 2.5 = 2.4$ (M1), $10^{8-3} = 10^5$, so 2.4×10^5 (A1)

CW9 (Indices, 2 marks)

$27^{\frac{1}{3}} = 3$ (M1)

$3^2 = 9$ (A1)

CW10 (Indices, 4 marks)

(a) $(\frac{16}{9})^{\frac{3}{2}} = (\frac{4}{3})^3$ (M1) $= \frac{64}{27}$ (A1)

(b) $2^{2x} = 2^{-3}$ (M1) $x = -\frac{3}{2}$ (A1)

CW11 (Surds, 2 marks)

$\sqrt{48} = \sqrt{16} \times \sqrt{3}$ (M1)

$4\sqrt{3}$ (A1)

CW12 (Surds, 4 marks)

(a) $\frac{10\sqrt{5}}{5}$ (M1) $= 2\sqrt{5}$ (A1)

(b) $15 - 3\sqrt{2} + 5\sqrt{2} - 2$ (M1) $= 13 + 2\sqrt{2}$ (A1)

CW13 (Surds, 6 marks)

(a) $5\sqrt{3} + 2\sqrt{3}$ (M1) $= 7\sqrt{3}$ (A1)

(b) $\frac{6(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{6(\sqrt{3}+1)}{2}$ (M1) $= 3 + 3\sqrt{3}$ (A1)

(c) $9 + 3\sqrt{3} + 3\sqrt{3} + 3$ (M1) $= 12 + 6\sqrt{3}$ (A1)

CW14 (Bounds, 2 marks)

Lower bound 6.35 (B1)

Upper bound 6.45 (B1)

CW15 (Bounds, 6 marks)

(a) UB length 12.55, UB width 8.25 (M1) 2(12.55 + 8.25) = 41.6 cm (A1)

(b) 12.45 × 8.15 (M1) = 101.4675 cm² (accept 101.47) (A1)

(c) UB length ÷ LB width = 12.55 ÷ 8.15 (M1) = 1.54 (A1)

Homework

HW1 (FDP, 2 marks)

$\frac{13}{4} \times \frac{8}{13}$ (M1)

2 (A1)

HW2 (FDP, 2 marks)

Decimals: 0.7, 0.625, 0.68, 0.666... (M1 for at least three correct conversions)

$\frac{5}{8}$, $\frac{2}{3}$, 68%, 0.7 (A1)

HW3 (FDP, 4 marks)

$\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$, so $\frac{4}{15}$ is left (M1 M1)

$\frac{3}{4} \times \frac{4}{15} = \frac{1}{5}$ of his pay is saved (M1)

84 × 5 = £420 (A1)

Check: rent 140, food 168, left 112, $\frac{3}{4}$ of 112 = 84. ✓

HW4 (FDP, 6 marks)

(a) $x = 0.4555\ldots$ $10x = 4.555\ldots$ $100x = 45.555\ldots$ (M1)

$100x - 10x = 41$, so $90x = 41$ (M1) $x = \frac{41}{90}$ (A1)

(b) $\frac{8}{3} - \frac{9}{5}$ (M1) $= \frac{40}{15} - \frac{27}{15}$ (M1) $= \frac{13}{15}$ (A1)

HW5 (Percentage change, 2 marks)

36×1.125 (M1)

£40.50 (A1)

HW6 (Percentage change, 2 marks)

$12\,400 - 10\,540 = 1860$ (M1)

$\frac{1860}{12\,400} \times 100 = 15\%$ (A1)

HW7 (Percentage change, 4 marks)

(a) $240\,000 \times 1.05 \times 0.97$ (M1) = £244 440 (A1)

(b) $244\,440 \div 240\,000 = 1.0185$ (M1) = 1.85% increase (A1)

HW8 (Percentage change, 4 marks)

(a) $18\,000 \times 1.025^3$ (M1) = 19 384 (A1)

(b) $1.04^2 = 1.0816$ (M1) = 8.16% increase (A1)

HW9 (Percentage change, 6 marks)

(a) $15\,000 \times 0.82 = 12\,300$ (M1) then $\times\ 0.88^2$ (M1) = £9525.12 (A1)

(b) Repeated × 0.88: year 4 £8382.11, year 5 £7376.25, year 6 £6491.10, year 7 £5712.17, year 8 £5026.71, year 9 £4423.50 (M1 for repeated multiplication, A1 for year 8 value above £5000)

9 years (A1)

HW10 (Reverse %, 2 marks)

$330 \div 1.2$ (M1)

£275 (A1)

HW11 (Reverse %, 4 marks)

(a) $31\,200 \div 1.04$ (M1) = £30 000 (A1)

(b) $31\,200 \times 1.025$ (M1) = £31 980 (A1)

HW12 (Reverse %, 6 marks)

(a) $84 \div 0.7$ (M1) = £120 (A1)

(b) Shop B normal price £125; 125×0.75 (M1) = £93.75 (A1)

(c) Shop A is cheaper (B1) by £93.75 − £84 = £9.75 (B1)

HW13 (Standard form, 2 marks)

0.00036 (B2). Award B1 for 0.0036 or 0.000036.

HW14 (Standard form, 2 marks)

28×10^4 (M1)

2.8×10^5 (A1)

HW15 (Standard form, 4 marks)

(a) $5.97 \times 10^{24} \div 7.35 \times 10^{22}$ (M1) = 81.2 (A1)

(b) $5.97 \times 10^{24} + 0.0735 \times 10^{24}$ (M1) $= 6.0435 \times 10^{24} = 6.04 \times 10^{24}$ kg (A1)

HW16 (Standard form, 6 marks)

(a) 4.5×10^9 km $= 4.5 \times 10^{12}$ m (B1)

$4.5 \times 10^{12} \div 3 \times 10^8$ (M1) = 15 000 (A1) $= 1.5 \times 10^4$ seconds (A1)

(b) 15 000 ÷ 60 = 250 minutes (M1) = 4 hours 10 minutes (A1)

HW17 (Indices, 2 marks)

$9x^8$ (M1)

$18x^5$ (A1)

HW18 (Indices, 2 marks)

$\frac{1}{8^{\frac{2}{3}}} = \frac{1}{4}$ (M1 for $8^{\frac{2}{3}} = 4$)

$\frac{1}{4}$ (A1)

HW19 (Indices, 4 marks)

(a) $3^x = 3^{-4}$ (M1) $x = -4$ (A1)

(b) $2^{3x-1} = 2^4$ (M1) $3x - 1 = 4$, so $x = \frac{5}{3}$ (A1)

HW20 (Indices, 4 marks)

(a) Square root of each part: $\frac{4x^4}{3y^{-2}}$ (M1) $= \frac{4x^4y^2}{3}$ (A1)

(b) $3^{2n} = 3^{3(n-1)}$ (M1) $2n = 3n - 3$, so $n = 3$ (A1)

HW21 (Surds, 2 marks)

$7\sqrt{2} - 4\sqrt{2}$ (M1)

$3\sqrt{2}$ (A1)

HW22 (Surds, 2 marks)

$5 - 2\sqrt{5} + 2\sqrt{5} - 4$ (M1) or use of difference of two squares

1 (A1)

HW23 (Surds, 4 marks)

(a) Multiply top and bottom by $(3 - \sqrt{5})$ (M1)

Denominator $9 - 5 = 4$ (M1) $\frac{12(3 - \sqrt{5})}{4} = 9 - 3\sqrt{5}$ (A1)

(b) $3\sqrt{5}$ (B1)

HW24 (Surds, 4 marks)

$h^2 = (3\sqrt{2})^2 + (\sqrt{14})^2$ (M1)

$= 18 + 14 = 32$ (M1 A1)

$h = \sqrt{32} = 4\sqrt{2}$ cm (A1)

HW25 (Surds, 6 marks)

(a) $\frac{4}{\sqrt{2}} = 2\sqrt{2}$ (M1) $\frac{\sqrt{8}}{\sqrt{2}} = \sqrt{4} = 2$ (M1) $2 + 2\sqrt{2}$ (A1)

(b) Multiply top and bottom by $(5 + \sqrt{2})$ (M1)

Numerator $(5 + \sqrt{2})^2 = 25 + 10\sqrt{2} + 2 = 27 + 10\sqrt{2}$ (M1)

Denominator $25 - 2 = 23$, so $\frac{27 + 10\sqrt{2}}{23}$ (A1)

HW26 (Bounds, 2 marks)

445 (B1)

$445 \leq m < 455$ (B1)

HW27 (Bounds, 2 marks)

3.15 − 1.65 (M1)

1.5 (A1)

HW28 (Bounds, 4 marks)

(a) 245 ÷ 7.95 (M1) = 30.8 m/s (A1)

(b) 235 ÷ 8.05 (M1) = 29.2 m/s (A1)

HW29 (Bounds, 4 marks)

UB radius 5.5 (B1)

UB height 12.5 (B1)

$\pi \times 5.5^2 \times 12.5$ (M1)

1188 cm^3 (A1) (1187.91...)

HW30 (Bounds, 6 marks)

(a) 3550 ÷ 45.5 (M1) = 78.02... (A1)

(b) 3650 ÷ 44.5 (M1) = 82.02... (A1)

(c) To 1 s.f. both bounds round to 80 (B1). To 2 s.f. they would be 78 and 82, which differ, so the length is 80 m to 1 s.f. (B1)