EDEXCEL GCSE (9-1) MATHEMATICS 1MA1 | HIGHER TIER | YEAR 11

Circle Theorems Answer Key

Detailed working and mark schemes for all classwork and homework questions

Tutor: Anjali

To be used with the Circle Theorems Lesson Workbook.

How to read the solutions

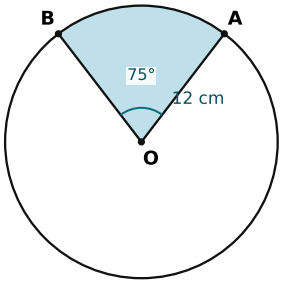
Each line is a step of working. [*Reasons in square brackets*] are the wording to write for the C mark. Mark codes: **(M1)** method, **(A1)** accuracy, **(B1)** independent, **(C1)** communication (reason).

Marks for reasons are only awarded when the reason matches the step. When marking, check three things on every line: the value, the theorem named, and whether the theorem actually applies to those angles.

Classwork Answers

CW1 | Sectors — Grade 4 | 4 marks

Question: The diagram shows a sector of a circle of radius 12 cm. The angle at the centre is 75°. (a) Work out the arc length of the sector. Give your answer in terms of π. [2] (b) Work out the area of the sector. Give your answer correct to 3 significant figures. [2]



(a) Fraction of circle = $\frac{75}{360} = \frac{5}{24}$. (M1 for $\frac{75}{360} \times 2 \times \pi \times 12$)

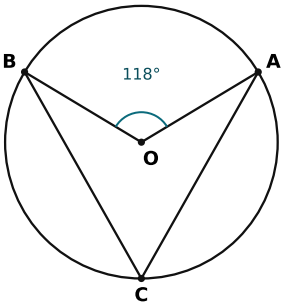
Arc length = $\frac{5}{24} \times 2\pi \times 12 = 5\pi$ cm **(A1)**

(b) Area = $\frac{75}{360} \times \pi \times 12^2 = \frac{5}{24} \times 144\pi = 30\pi$ **(M1)**

$30\pi = 94.24... = 94.2$ cm² (3 s.f.) **(A1)**

CW2 | Angle at the centre — Grade 4 | 2 marks

Question: A, B and C are points on the circumference of a circle with centre O. Angle AOB = 118°. Work out the size of angle ACB. Give a reason for your answer.

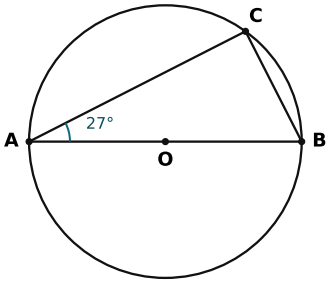


Angle ACB = 118° ÷ 2 = 59° **(A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*] **(C1)**

CW3 | Angle in a semicircle — Grade 4 | 2 marks

Question: AB is a diameter of the circle. C is a point on the circumference. Angle CAB = 27°. Work out the size of angle ABC. Give reasons for your answer.

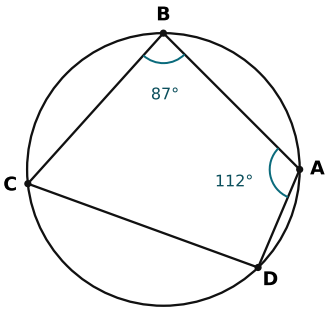


Angle ACB = 90° [*angle in a semicircle is 90°*] (M1 with reason)

Angle ABC = 180° − 90° − 27° = 63° [*angles in a triangle add up to 180°*] **(A1)**

CW4 | Cyclic quadrilateral — Grade 5 | 4 marks

Question: A, B, C and D are points on a circle. Angle DAB = 112° and angle ABC = 87°. (a) Work out the size of angle BCD. Give a reason. [2] (b) Work out the size of angle ADC. Give a reason. [2]



(a) Angle BCD = 180° − 112° = 68° **(A1)**

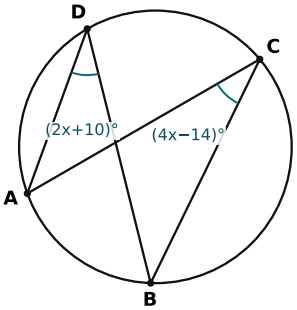
Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*] **(C1)**

(b) Angle ADC = 180° − 87° = 93° **(A1)**

Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*] **(C1)**

CW5 | Angles in the same segment — Grade 6 | 3 marks

Question: A, B, C and D are points on a circle. Angle ADB = (2x + 10)° and angle ACB = (4x − 14)°. Work out the value of x and the size of angle ADB. Give a reason.



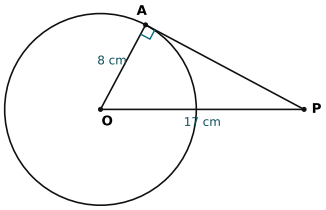
Angle ADB = angle ACB [*angles in the same segment are equal*] **(M1)**

2x + 10 = 4x − 14, so 24 = 2x and x = 12 **(A1)**

Angle ADB = 2 × 12 + 10 = 34°. Check: 4 × 12 − 14 = 34°. ✓ **(A1)**

CW6 | Tangent and radius — Grade 6 | 3 marks

Question: PA is a tangent to a circle with centre O. A is the point of contact. The radius of the circle is 8 cm and OP = 17 cm. Work out the length of PA.



Angle OAP = 90° [*the angle between a tangent and a radius is 90°*]

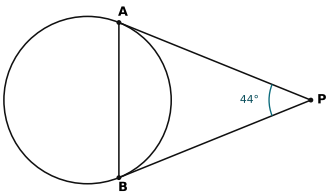
$PA^2 = OP^2 - OA^2 = 17^2 - 8^2$ (M1 Pythagoras with the correct hypotenuse)

= 289 − 64 = 225 **(M1)**

PA = 15 cm **(A1)**

CW7 | Two tangents — Grade 6 | 3 marks

Question: PA and PB are tangents to a circle. Angle APB = 44°. Work out the size of angle PAB. Give reasons for your answer.



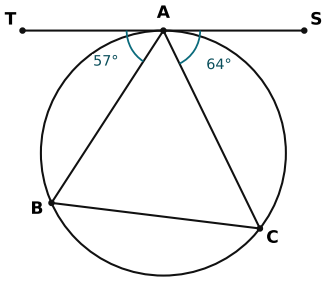
PA = PB [*tangents from an external point are equal in length*] **(C1)**

So triangle PAB is isosceles. Angle PAB = (180° − 44°) ÷ 2 **(M1)**

= 68° **(A1)**

CW8 | Alternate segment theorem — Grade 7 | 4 marks

Question: TAS is a tangent to the circle at A. Angle TAB = 57° and angle SAC = 64°. (a) Work out the size of angle ACB. Give a reason. [2] (b) Work out the size of angle BAC. [2]



(a) Angle ACB = angle TAB = 57° **(A1)**

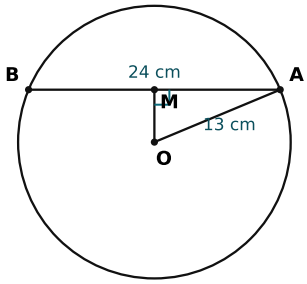
Reason: [*the angle between a tangent and a chord is equal to the angle in the alternate segment*] **(C1)**

(b) Angle BAC = 180° − 57° − 64° **(M1)**

= 59° [*angles on a straight line add up to 180°*] **(A1)**

CW9 | Chords — Grade 7 | 3 marks

Question: AB is a chord of a circle with centre O and radius 13 cm. AB = 24 cm. M is the midpoint of AB. Work out the distance OM.



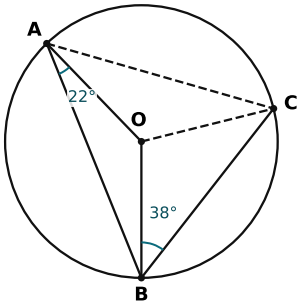
OM is perpendicular to AB and AM = 12 cm [*the line from the centre to the midpoint of a chord is perpendicular to the chord*] **(C1)**

$OM^2 = OA^2 - AM^2 = 13^2 - 12^2 = 169 - 144 = 25$ **(M1)**

OM = 5 cm **(A1)**

CW10 | Centre theorem and isosceles triangles — Grade 7 | 4 marks

Question: A, B and C are points on a circle with centre O. Angle OAB = 22° and angle OBC = 38°. (a) Work out the size of angle ABC. Give reasons. [2] (b) Work out the size of angle AOC. Give a reason. [2]



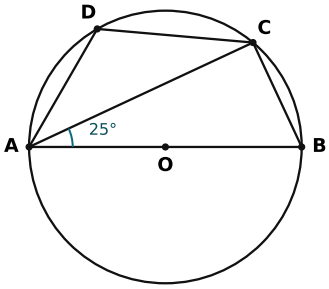
(a) OA = OB (radii), so angle OBA = angle OAB = 22° [*base angles of an isosceles triangle are equal*] **(M1)**

Angle ABC = 22° + 38° = 60° **(A1)**

(b) Angle AOC = 2 × 60° = 120° [*the angle at the centre is twice the angle at the circumference*] **(M1 A1)**

CW11 | Semicircle and cyclic quadrilateral — Grade 7 | 4 marks

Question: AB is a diameter of the circle with centre O. A, B, C and D are points on the circle. Angle CAB = 25°. (a) Write down the size of angle ACB. Give a reason. [1] (b) Work out the size of angle ABC. [1] (c) Work out the size of angle ADC. Give a reason. [2]



(a) 90° [*angle in a semicircle is 90°*] **(B1)**

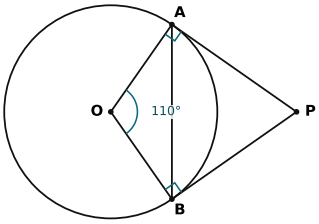
(b) Angle ABC = 180° − 90° − 25° = 65° [*angles in a triangle add up to 180°*] **(B1)**

(c) Angle ADC = 180° − 65° = 115° **(M1 A1)**

Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*]

CW12 | Tangents and the kite — Grade 8 | 5 marks

Question: A and B are points on a circle with centre O. PA and PB are tangents to the circle. Angle AOB = 110°. (a) Work out the size of angle APB. Give reasons. [2] (b) Work out the size of angle PAB. Give a reason. [2] (c) Work out the size of angle OAB. [1]



(a) Angles OAP and OBP = 90° [*tangent perpendicular to radius*]. Angle APB = 360° − 90° − 90° − 110° **(M1)** = 70° [*angles in a quadrilateral add up to 360°*] **(A1)**

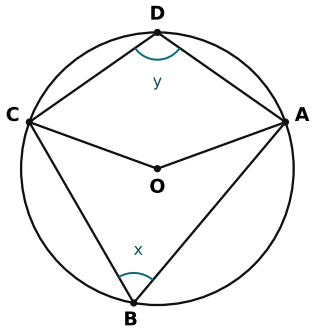
(b) PA = PB [*tangents from an external point are equal in length*], so angle PAB = (180° − 70°) ÷ 2 = 55° **(M1 A1)**

(c) Angle OAB = 90° − 55° = 35° **(B1)**

Check using triangle OAB (isosceles, OA = OB): (180° − 110°) ÷ 2 = 35°. ✓

CW13 | Proof: cyclic quadrilateral — Grade 9 | 5 marks

Question: A, B, C and D are points on a circle with centre O. Angle ABC = x and angle ADC = y. Prove that x + y = 180°.



Let angle AOC (the angle on the side of D, less than 180°) = z. Then the reflex angle AOC = 360° − z [*angles round a point add up to 360°*] **(M1)**

$x = \frac{z}{2}$ [*the angle at the centre is twice the angle at the circumference*] **(B1, C1)**

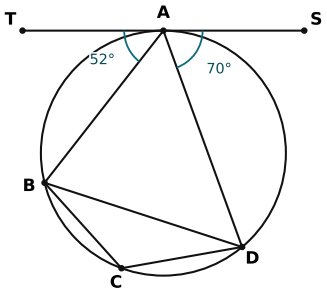
$y = \frac{360° - z}{2}$ [*the angle at the centre is twice the angle at the circumference*] **(B1, C1)**

$x + y = \frac{z}{2} + \frac{360° - z}{2} = \frac{360°}{2} = 180°$ **(A1)**

So opposite angles of a cyclic quadrilateral add up to 180°. ∎

CW14 | Tangent, cyclic quadrilateral and alternate segment — Grade 8 | 6 marks

Question: TAS is a tangent to the circle at A. B, C and D are points on the circle. Angle TAB = 52° and angle SAD = 70°. (a) Work out the size of angle BAD. Give a reason. [2] (b) Work out the size of angle BCD. Give a reason. [2] (c) Work out the size of angle ADB. Give a reason. [2]



(a) Angle BAD = 180° − 52° − 70° = 58° [*angles on a straight line add up to 180°*] **(M1 A1)**

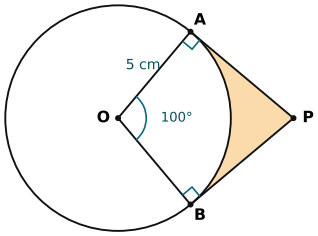
(b) Angle BCD = 180° − 58° = 122° [*opposite angles of a cyclic quadrilateral add up to 180°*] **(M1 A1)**

(c) Angle ADB = angle TAB = 52° **(A1)**

Reason: [*alternate segment theorem*] **(C1)**

CW15 | Tangents with areas — Grade 9 | 6 marks

Question: PA and PB are tangents to a circle with centre O and radius 5 cm. Angle AOB = 100°. Work out the area of the shaded region, bounded by PA, PB and the minor arc AB. Give your answer correct to 3 significant figures.



Angle OAP = 90° [*tangent perpendicular to radius*] and angle AOP = 100° ÷ 2 = 50° [*kite OAPB is symmetrical about OP*] **(M1)**

$\tan 50° = \frac{AP}{5}$, so AP = 5 tan 50° = 5.958... **(M1 A1)**

Area of kite OAPB = $2 \times \frac{1}{2} \times 5 \times 5.958... = 29.79...$ cm² **(M1)**

Area of sector OAB = $\frac{100}{360} \times \pi \times 5^2 = 21.81...$ cm² **(M1)**

Shaded area = 29.79... − 21.81... = 7.977... = 7.98 cm² (3 s.f.) **(A1)**

Homework Answers

HW1 | Circle measures — Grade 4 | 2 marks

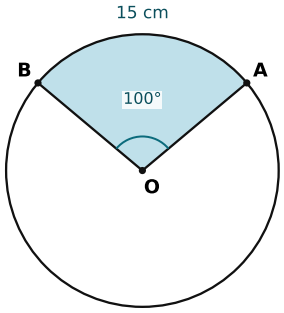
Question: The circumference of a circle is 18π cm. Work out the area of the circle. Give your answer in terms of π.

$2\pi r = 18\pi$, so $r = 9$ cm **(M1)**

Area $= \pi \times 9^2 = 81\pi$ cm² **(A1)**

HW2 | Sectors — Grade 5 | 3 marks

Question: The diagram shows a sector of a circle. The angle at the centre is 100° and the arc length is 15 cm. Work out the radius of the circle. Give your answer correct to 3 significant figures.



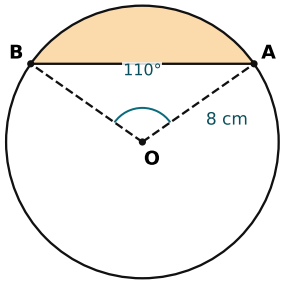
Arc length $= \frac{100}{360} \times 2\pi r = \frac{5\pi r}{9}$ **(M1)**

$\frac{5\pi r}{9} = 15$, so $r = \frac{135}{5\pi} = \frac{27}{\pi}$ **(M1)**

$r = 8.59$ cm (3 s.f.) **(A1)**

HW3 | Segments — Grade 7 | 4 marks

Question: The diagram shows a circle of radius 8 cm with centre O. The chord AB subtends an angle of 110° at O. Work out the area of the shaded segment. Give your answer correct to 3 significant figures.



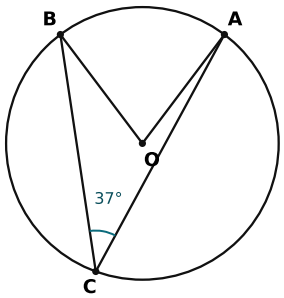
Sector OAB $= \frac{110}{360} \times \pi \times 8^2 = 61.43...$ cm² **(M1 A1)**

Triangle OAB $= \frac{1}{2} \times 8 \times 8 \times \sin 110° = 30.07...$ cm² **(M1)**

Segment $= 61.43... - 30.07... = 31.36... = 31.4$ cm² (3 s.f.) **(A1)**

HW4 | Angle at the centre — Grade 4 | 2 marks

Question: A, B and C are points on a circle with centre O. Angle ACB = 37°. Work out the size of angle AOB. Give a reason.

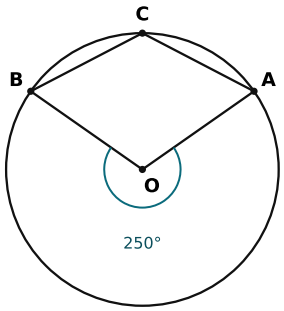


Angle AOB $= 2 \times 37° = 74°$ **(A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*] **(C1)**

HW5 | Angle at the centre (reflex) — Grade 7 | 3 marks

Question: A, B and C are points on a circle with centre O. C lies on the minor arc AB. The reflex angle AOB is 250°. Work out the size of angle ACB. Give a reason.



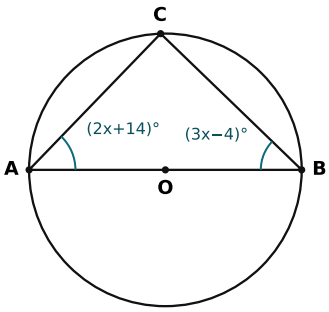
Angle ACB = 250° ÷ 2 **(M1)**

= 125° **(A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*] **(C1)**

HW6 | Angle in a semicircle — Grade 6 | 3 marks

Question: AB is a diameter of a circle with centre O. C is a point on the circle. Angle BAC = (2x + 14)° and angle ABC = (3x − 4)°. Work out the value of x. Give a reason.



Angle ACB = 90° [*angle in a semicircle is 90°*]

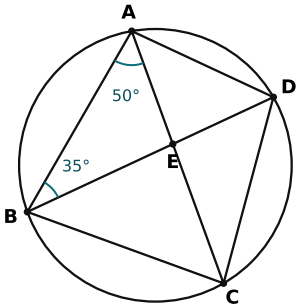
(2x + 14) + (3x − 4) + 90 = 180 [*angles in a triangle add up to 180°*] **(M1)**

5x + 100 = 180, so 5x = 80 **(A1)**

x = 16 **(A1)** [Check: angle BAC = 46°, angle ABC = 44°, 46 + 44 + 90 = 180. ✓]

HW7 | Angles in the same segment — Grade 6 | 4 marks

Question: A, B, C and D are points on a circle. The chords AC and BD intersect at E. Angle ABD = 35° and angle BAC = 50°. (a) Write down the size of angle ACD. Give a reason. [2] (b) Work out the size of angle AEB. [2]



(a) Angle ACD = angle ABD = 35° **(A1)**

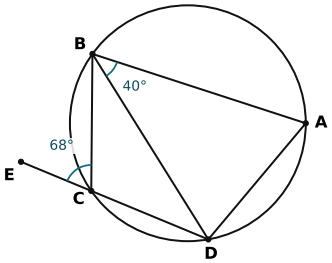
Reason: [*angles in the same segment are equal*] **(C1)**

(b) Angle AEB = 180° − 50° − 35° **(M1)**

= 95° [*angles in a triangle add up to 180°*] **(A1)**

HW8 | Cyclic quadrilateral (exterior angle) — Grade 6 | 3 marks

Question: ABCD is a cyclic quadrilateral. DC is extended to E. Angle BCE = 68° and angle ABD = 40°. (a) Work out the size of angle BAD. Give a reason. [2] (b) Work out the size of angle ADB. [1]



(a) Angle BAD = angle BCE = 68° **(A1)**

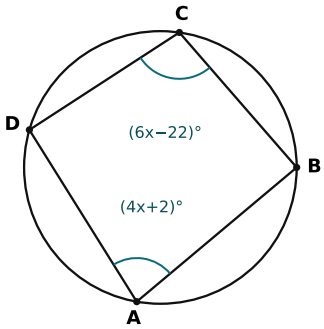
Reason: [*the exterior angle of a cyclic quadrilateral equals the interior opposite angle*] **(C1)**

(b) Angle ADB = 180° − 68° − 40° = 72° [*angles in a triangle add up to 180°*] **(B1)**

HW9 | Cyclic quadrilateral (algebra)

Grade 7 | 4 marks

Question: A, B, C and D are points on a circle. Angle DAB = (4x + 2)° and angle BCD = (6x − 22)°. (a) Work out the value of x. Give a reason for your first step. [3] (b) Write down the size of angle DAB. [1]



(a) (4x + 2) + (6x − 22) = 180 [*opposite angles of a cyclic quadrilateral add up to 180°*] **(M1)**

10x − 20 = 180, so 10x = 200 **(M1)**

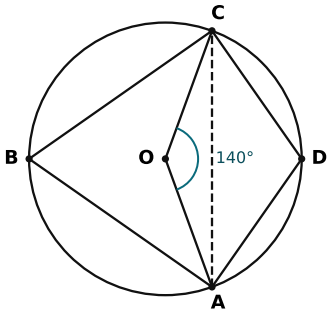
x = 20 **(A1)**

(b) Angle DAB = 4 × 20 + 2 = 82° **(B1)** [Check: angle BCD = 6 × 20 − 22 = 98°, and 82 + 98 = 180. ✓]

HW10 | Centre theorem and cyclic quadrilateral

Grade 8 | 5 marks

Question: A, B, C and D are points on a circle with centre O. Angle AOC = 140°. B is on the major arc AC and D is on the minor arc AC. (a) Work out the size of angle ABC. Give a reason. [2] (b) Work out the size of angle ADC. Give a reason. [2] (c) Work out the size of angle OCA. [1]



(a) Angle ABC = 140° ÷ 2 = 70° **(A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*] **(C1)**

(b) Angle ADC = 180° − 70° = 110° **(A1)**

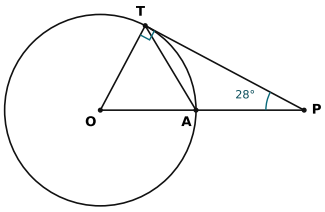
Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*] **(C1)**

(c) Triangle OAC is isosceles (OA = OC, radii). Angle OCA = (180° − 140°) ÷ 2 = 20° **(B1)**

HW11 | Tangent and isosceles triangle

Grade 8 | 4 marks

Question: PT is a tangent to a circle with centre O at the point T. The line PO meets the circle at A. Angle TPO = 28°. Work out the size of angle ATP. You must show all your working.



Angle OTP = 90° [*the angle between a tangent and a radius is 90°*]

Angle TOP = 180° − 90° − 28° = 62° [*angles in a triangle add up to 180°*] **(M1)**

Triangle OTA is isosceles (OT = OA, radii), so angle OTA = (180° − 62°) ÷ 2 = 59° **(M1)**

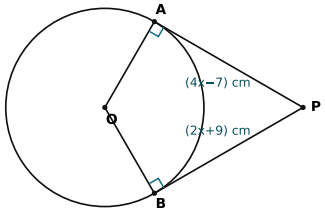
Angle ATP = angle OTP − angle OTA = 90° − 59° **(M1)**

= 31° **(A1)**

HW12 | Two tangents (algebra)

Grade 7 | 3 marks

Question: PA and PB are tangents to a circle. PA = (4x − 7) cm and PB = (2x + 9) cm. Work out the length of PA. Give a reason for your first step.



PA = PB [*tangents from an external point are equal in length*]

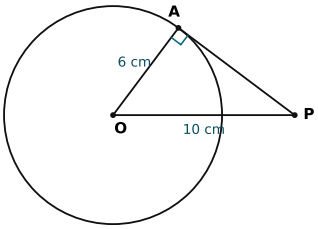
4x − 7 = 2x + 9, so 2x = 16 **(M1)**

x = 8 **(A1)**

PA = 4 × 8 − 7 = 25 cm **(A1)** [Check: PB = 2 × 8 + 9 = 25 cm. ✓]

HW13 | Tangent with Pythagoras and trigonometry — Grade 7 | 4 marks

Question: PA is a tangent to a circle with centre O at the point A. The radius of the circle is 6 cm and OP = 10 cm. (a) Work out the length of PA. [2] (b) Work out the size of angle APO. Give your answer correct to 1 decimal place. [2]



(a) Angle OAP = 90° [*tangent perpendicular to radius*]. $PA^2 = 10^2 - 6^2 = 100 - 36 = 64$ **(M1)**

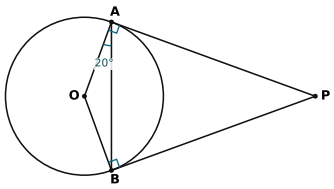
PA = 8 cm **(A1)**

(b) $\sin(APO) = \frac{OA}{OP} = \frac{6}{10}$ **(M1)**

Angle APO = $\sin^{-1}(0.6) = 36.9°$ (1 d.p.) **(A1)** [Alternative: $\cos^{-1}(\frac{8}{10})$ or $\tan^{-1}(\frac{6}{8})$.]

HW14 | Tangents and isosceles triangles — Grade 8 | 5 marks

Question: PA and PB are tangents to a circle with centre O. Angle OAB = 20°. (a) Work out the size of angle APB. Give reasons. [3] (b) Work out the size of angle PAB. [2]



(a) OA = OB (radii), so angle OBA = 20° and angle AOB = 180° − 20° − 20° = 140° [*base angles; angles in a triangle*] **(M1)**

Angles OAP and OBP = 90° [*tangent perpendicular to radius*]. Angle APB = 360° − 90° − 90° − 140° **(M1)**

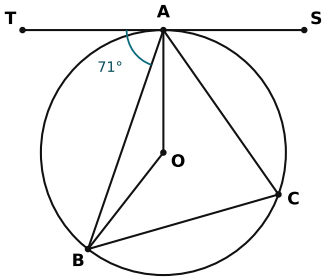
= 40° [*angles in a quadrilateral add up to 360°*] **(A1)**

(b) Angle PAB = angle OAP − angle OAB = 90° − 20° **(M1)** = 70° **(A1)**

Check: PA = PB, so angle PAB = (180° − 40°) ÷ 2 = 70°. ✓

HW15 | Alternate segment and centre theorem — Grade 7 | 3 marks

Question: TAS is a tangent to a circle with centre O. A, B and C are points on the circle. Angle TAB = 71°. (a) Write down the size of angle ACB. Give a reason. [1] (b) Work out the size of angle AOB. Give a reason. [2]



(a) Angle ACB = 71° [*alternate segment theorem*] **(B1)**

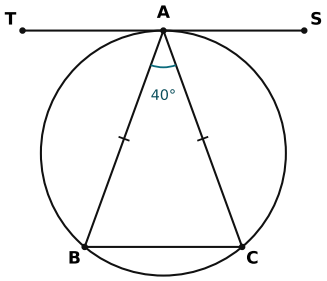
(b) Angle AOB = 2 × 71° = 142° **(M1 A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*]

Check: triangle OAB is isosceles, so angle OAB = (180° − 142°) ÷ 2 = 19°, and angle TAB + ... the tangent-radius angle gives 71° + 19° = 90°. ✓

HW16 | Alternate segment and isosceles triangle — Grade 8 | 4 marks

Question: A, B and C are points on a circle. AB = AC and angle BAC = 40°. TAS is the tangent to the circle at A. (a) Work out the size of angle ABC. [2] (b) Work out the size of angle TAB. Give a reason. [2]



(a) Angle ABC = (180° − 40°) ÷ 2 = 70° **(M1 A1)** [*base angles of an isosceles triangle are equal*]

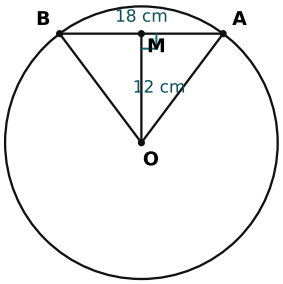
(b) Angle ACB = 70°, and angle TAB = angle ACB = 70° **(A1)**

Reason: [*alternate segment theorem*] **(C1)**

Check: angle TAB = angle ABC = 70°. These are alternate angles, so TAS is parallel to BC. ✓

HW17 | Chords and the centre — Grade 8 | 4 marks

Question: AB is a chord of a circle with centre O. M is the midpoint of AB. AB = 18 cm and OM = 12 cm. (a) Work out the radius of the circle. [2] (b) Work out the size of angle AOB. Give your answer correct to 1 decimal place. [2]



(a) AM = 9 cm and angle OMA = 90° [*the line from the centre to the midpoint of a chord is perpendicular to the chord*]

$OA^2 = 12^2 + 9^2 = 144 + 81 = 225$ **(M1)**

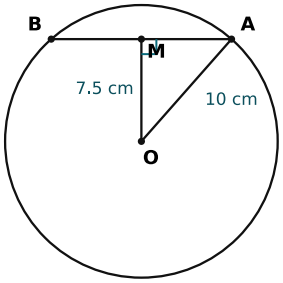
OA = 15 cm **(A1)**

(b) $\sin(AOM) = \frac{9}{15}$ **(M1)**

Angle AOM = $\sin^{-1}(0.6) = 36.87...°$, so angle AOB = $2 \times 36.87...° = 73.7°$ (1 d.p.) **(A1)**

HW18 | Chords and Pythagoras — Grade 7 | 3 marks

Question: A chord AB is drawn in a circle with centre O and radius 10 cm. The perpendicular distance from O to AB is 7.5 cm. Work out the length of AB. Give your answer correct to 3 significant figures.



Let M be the midpoint of AB. Angle OMA = 90° and AM = $\frac{1}{2}$AB [*perpendicular from the centre bisects the chord*]

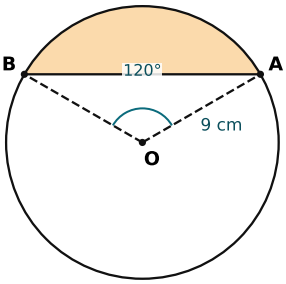
$AM^2 = 10^2 - 7.5^2 = 100 - 56.25 = 43.75$ **(M1)**

AM = 6.614... (M1 for AB = 2 × AM)

AB = 13.2 cm (3 s.f.) **(A1)**

HW19 | Segments — Grade 8 | 5 marks

Question: AB is a chord of a circle with centre O and radius 9 cm. Angle AOB = 120°. The shaded region is a segment. (a) Work out the area of sector OAB, in cm². Give your answer to 3 significant figures. [2] (b) Work out the area of triangle OAB. [2] (c) Work out the area of the shaded segment. [1]



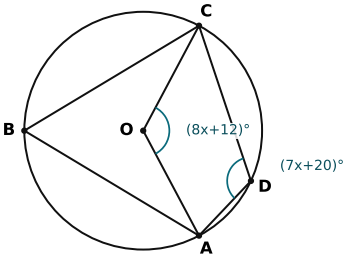
(a) $\frac{120}{360} \times \pi \times 9^2 = 27\pi$ **(M1)** $= 84.8$ cm² (3 s.f.) **(A1)**

(b) $\frac{1}{2} \times 9 \times 9 \times \sin 120°$ **(M1)** $= 35.07... = 35.1$ cm² **(A1)**

(c) Segment = 84.82... − 35.07... = 49.7 cm² (3 s.f.) **(B1)**

HW20 | Algebra with several theorems — Grade 9 | 6 marks

Question: A, B, C and D are points on a circle with centre O. B is on the major arc AC and D is on the minor arc AC. Angle AOC = (8x + 12)° and angle ADC = (7x + 20)°. (a) Work out the value of x. Give reasons for each step. [4] (b) Work out the size of the reflex angle AOC. [2]



(a) Angle ABC = $\frac{1}{2}(8x + 12) = (4x + 6)°$ [*the angle at the centre is twice the angle at the circumference*] **(M1)**

Angle ADC = 180° − (4x + 6) = (174 − 4x)° [*opposite angles of a cyclic quadrilateral add up to 180°*] **(M1)**

7x + 20 = 174 − 4x, so 11x = 154 **(M1)**

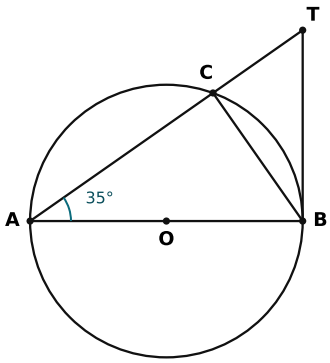
x = 14 **(A1)**

(b) Angle AOC = 8 × 14 + 12 = 124°. Reflex angle AOC = 360° − 124° **(M1)** = 236° [*angles round a point add up to 360°*] **(A1)**

Check: angle ADC = 7 × 14 + 20 = 118°, angle ABC = 62°, and 118 + 62 = 180. ✓

HW21 | Tangent and semicircle — Grade 8 | 5 marks

Question: AB is a diameter of a circle with centre O. C is a point on the circle. BT is a tangent to the circle at B. AC extended meets BT at T. Angle BAC = 35°. (a) Write down the size of angle ACB. Give a reason. [1] (b) Work out the size of angle CBT. Give reasons. [2] (c) Work out the size of angle ATB. [2]



(a) Angle ACB = 90° [*angle in a semicircle is 90°*] **(B1)**

(b) Angle ABC = 180° − 90° − 35° = 55° [*angles in a triangle add up to 180°*]

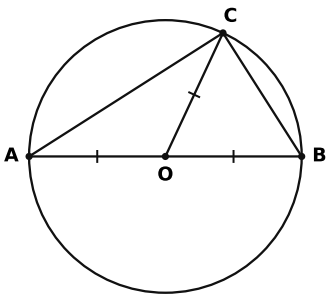
Angle ABT = 90° [*the angle between a tangent and a radius (here the diameter) is 90°*]

Angle CBT = 90° − 55° = 35° **(M1 A1)** [Alternate segment theorem gives the same: angle CBT = angle BAC = 35°.]

(c) In triangle ABT: angle BAT = 35°, angle ABT = 90°, so angle ATB = 180° − 35° − 90° = 55° **(M1 A1)**

HW22 | Proof: angle in a semicircle — Grade 8 | 3 marks

Question: A, B and C are points on a circle with centre O. AB is a diameter. Prove that angle ACB = 90°.



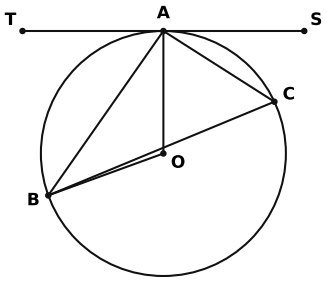
Let angle OAC = a. OA = OC (radii), so angle OCA = a [*base angles of an isosceles triangle are equal*]. Let angle OBC = b. OB = OC, so angle OCB = b. **(M1)**

In triangle ABC: a + b + (a + b) = 180° [*angles in a triangle add up to 180°*], so 2(a + b) = 180° and a + b = 90° **(M1)**

Angle ACB = angle OCA + angle OCB = a + b = 90°. ∎ **(A1)**

HW23 | Proof: alternate segment theorem — Grade 9 | 6 marks

Question: TAS is a tangent to a circle with centre O at A. B and C are points on the circle. Prove that angle TAB = angle ACB. (You may add a construction to the diagram.)



Draw the diameter AD and join BD. Let angle TAB = y. **(B1)**

Angle TAD = 90° [*a tangent is perpendicular to the radius*], so angle DAB = 90° − y **(C1)**

Angle ABD = 90° [*angle in a semicircle is 90°*] **(C1)**

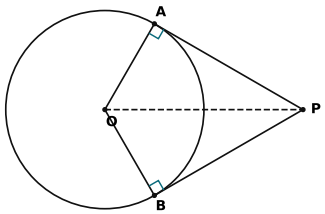
Angle ADB = 180° − 90° − (90° − y) = y [*angles in a triangle add up to 180°*] **(M1)**

Angle ACB = angle ADB [*angles in the same segment are equal*] **(C1)**

So angle ACB = y = angle TAB. ∎ **(A1)**

HW24 | Proof: equal tangents — Grade 8 | 4 marks

Question: PA and PB are tangents to a circle with centre O. Prove that PA = PB.



Consider triangles OAP and OBP.

OA = OB [*radii of the same circle*] **(B1)**

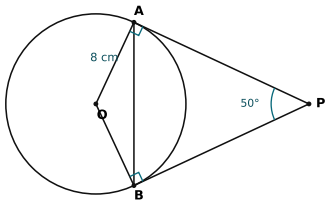
Angle OAP = angle OBP = 90° [*a tangent is perpendicular to the radius*] **(B1)**

OP is a common side **(B1)**

The triangles are congruent (RHS: right angle, hypotenuse, side), so PA = PB. ∎ **(C1)**

HW25 | Tangents and trigonometry — Grade 9 | 5 marks

Question: PA and PB are tangents to a circle with centre O and radius 8 cm. Angle APB = 50°. (a) Work out the size of angle AOB. Give reasons. [2] (b) Work out the length of the chord AB. Give your answer correct to 3 significant figures. [3]



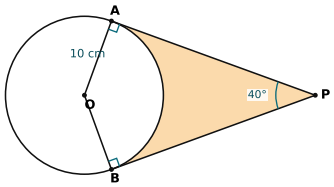
(a) Angles OAP and OBP = 90° [*tangent perpendicular to radius*]. Angle AOB = 360° − 90° − 90° − 50° **(M1)** = 130° [*angles in a quadrilateral*] **(A1)**

(b) Method 1: triangle OAB is isosceles. Half of angle AOB = 65°. AB = 2 × 8 × sin 65° **(M1 M1)** = 14.5 cm (3 s.f.) **(A1)**

Method 2 (cosine rule): $AB^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 130° = 210.3...$, so AB = 14.5 cm.

HW26 | Tangents with areas — Grade 9 | 6 marks

Question: PA and PB are tangents to a circle with centre O and radius 10 cm. Angle APB = 40°. Work out the area of the shaded region, bounded by PA, PB and the minor arc AB. Give your answer correct to 3 significant figures.



Angle AOB = 360° − 90° − 90° − 40° = 140°, so angle AOP = 70° [*tangent perpendicular to radius; angles in a quadrilateral; kite symmetry*] **(M1)**

$\tan 70° = \frac{AP}{10}$, so AP = 10 tan 70° = 27.47... **(M1 A1)**

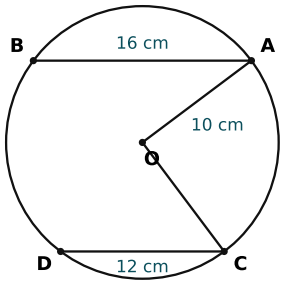
Area of kite OAPB = $2 \times \frac{1}{2} \times 10 \times 27.47... = 274.7...$ cm² **(M1)**

Area of sector OAB = $\frac{140}{360} \times \pi \times 10^2 = 122.1...$ cm² **(M1)**

Shaded area = 274.7... − 122.1... = 152.5... = 153 cm² (3 s.f.) **(A1)**

HW27 | Parallel chords — Grade 8 | 4 marks

Question: AB and CD are parallel chords on opposite sides of the centre O of a circle of radius 10 cm. AB = 16 cm and CD = 12 cm. Work out the perpendicular distance between the chords.



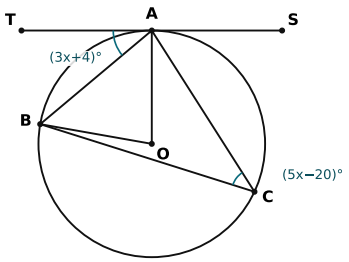
Distance from O to AB: the perpendicular from O bisects AB, so half of AB = 8 cm. Distance = $\sqrt{(10^2 - 8^2)} = \sqrt{36} = 6$ cm **(M1 A1)**

Distance from O to CD: half of CD = 6 cm. Distance = $\sqrt{(10^2 - 6^2)} = \sqrt{64} = 8$ cm **(M1)**

The chords are on opposite sides of O, so the distance between them = 6 + 8 = 14 cm **(A1)**

HW28 | Alternate segment (algebra) — Grade 8 | 5 marks

Question: TAS is a tangent to a circle with centre O at A. Angle TAB = (3x + 4)° and angle ACB = (5x − 20)°. (a) Work out the value of x. Give a reason. [3] (b) Work out the size of angle AOB. Give a reason. [2]



(a) Angle TAB = angle ACB [*alternate segment theorem*] **(C1)**

3x + 4 = 5x − 20, so 24 = 2x **(M1)**

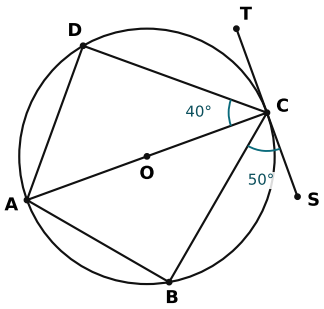
x = 12 **(A1)**

(b) Angle ACB = 5 × 12 − 20 = 40°. Angle AOB = 2 × 40° = 80° **(M1 A1)**

Reason: [*the angle at the centre is twice the angle at the circumference*]

HW29 | Mixed: tangent, semicircle, cyclic quadrilateral — Grade 9 | 6 marks

Question: AC is a diameter of a circle with centre O. A, B, C and D are points on the circle. ST is the tangent to the circle at C. Angle SCB = 50° and angle ACD = 40°. (a) Work out the size of angle BAC. Give a reason. [2] (b) Work out the size of angle CAD. Give reasons. [2] (c) Work out the size of angle BCD. Give a reason. [2]



(a) Angle BAC = angle SCB = 50° **(A1)**

Reason: [*alternate segment theorem*] **(C1)**

(b) Angle ADC = 90° [*angle in a semicircle is 90°*] **(M1)**

Angle CAD = 180° − 90° − 40° = 50° [*angles in a triangle add up to 180°*] **(A1)**

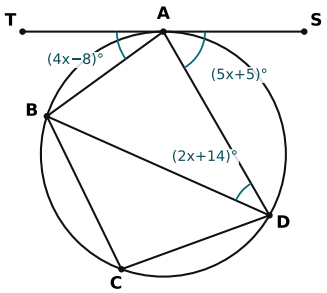
(c) Angle BAD = 50° + 50° = 100°. Angle BCD = 180° − 100° = 80° **(M1 A1)**

Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*]

Check by another route: angle ACB = 90° − 50° = 40° (tangent perpendicular to radius), and 40° + angle ACD (40°) = 80°. ✓

HW30 | Mixed: tangent, algebra, cyclic quadrilateral — Grade 9 | 6 marks

Question: TAS is a tangent to the circle at A. B, C and D are points on the circle. Angle TAB = (4x − 8)°, angle ADB = (2x + 14)° and angle SAD = (5x + 5)°. (a) Work out the value of x. Give a reason. [3] (b) Work out the size of angle BAD. [1] (c) Work out the size of angle BCD. Give a reason. [2]



(a) Angle TAB = angle ADB [*alternate segment theorem*] **(C1)**

4x − 8 = 2x + 14, so 2x = 22 **(M1)**

x = 11 **(A1)**

(b) Angle TAB = 36° and angle SAD = 5 × 11 + 5 = 60°. Angle BAD = 180° − 36° − 60° = 84° [*angles on a straight line add up to 180°*] **(B1)**

(c) Angle BCD = 180° − 84° = 96° **(M1 A1)**

Reason: [*opposite angles of a cyclic quadrilateral add up to 180°*]