EDEXCEL GCSE (9-1) MATHEMATICS 1MA1 | HIGHER TIER | YEAR 11

# Circle Theorems Lesson Workbook

Target grade 9: full explanations, proofs, worked examples, 15 classwork and 30 homework questions

## Tutor: Anjali

Answers with detailed working are in the separate Answer Key.

**Inside this workbook**


- Parts of a circle, sectors and segments
- Angle toolkit and how to write reasons
- Theorem 1: Angle at the centre
- Theorem 2: Angle in a semicircle
- Theorem 3: Same segment
- Theorem 4: Cyclic quadrilaterals
- Theorem 5: Tangent and radius
- Theorem 6: Two tangents
- Theorem 7: Alternate segment
- Theorem 8: Chords
- Areas, proofs and exam-style questions
- Reason bank and revision sheet
- Classwork and homework


Student name: _____

Class / date started: _____

Target grade: _____

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## Topic objectives checklist

| Objective | F/H | Where | Not yet / Getting there / Secure |
|---|---|---|---|
| Review the names of the parts of a circle. | F | Section 1 | |
| Review the area and circumference of circles and sectors. | F | Section 1 | |
| Recognise and apply circle theorems. | H | Sections 3 to 10 | |
| Solve circle problems and state the angle rules or circle theorems used to get the answer. | H | Sections 2, 11 to 13 | |

## How to use this workbook

1. **Learn the theorem** from its box, the picture and the exact reason wording.
2. **Read the proof**. At grade 9 you are expected to prove the main theorems, and understanding the proof is the best way to remember them.
3. **Cover the solution** of each worked example and try it first.
4. **Classwork** is done with your tutor. **Homework** is done alone, then marked against the Answer Key with a different colour pen.

5. Record scores in the trackers and redo any question where you lost the reason mark.

**About the exam-style questions.** Every question here is original, written to match the style, command words, mark allocation and difficulty of Edexcel 1MA1 Higher papers (Paper 1H non-calculator, Papers 2H and 3H calculator). They are not copied from past papers. Real past papers and mark schemes from 2018 onwards are free on the Pearson Edexcel website, and doing the circle theorem questions from them after this workbook is the right next step.

**Diagrams.** The diagrams in this workbook are drawn accurately so that you can see the geometry clearly. In the real exam they are marked 'Diagram NOT accurately drawn', so never measure an angle or assume a length. Use only what the question states and the theorems you can name.

## Mark scheme codes

| Code | Meaning |
|---|---|
| M1 | Method mark: a correct method is shown, even if the arithmetic slips. |
| A1 | Accuracy mark: a correct answer, usually only awarded if the method mark was earned. |
| B1 | Independent mark: a correct statement or value without needing a method. |
| C1 | Communication mark: a correct, named reason or a correctly structured argument. |

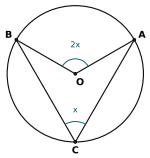
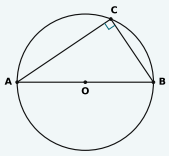
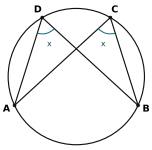
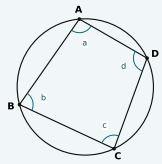
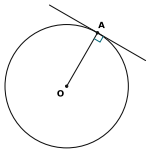
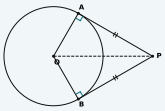
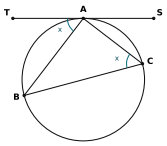
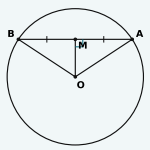
## What separates a grade 8 from a grade 9 here

- Grade 9 questions chain three or four theorems. Missing one reason in a chain costs marks at every later step.
- Algebra is added: expressions like (3x + 4)° must be set equal using a named theorem.
- Proofs: you must define letters, give a reason per line and finish with the statement you set out to prove.
- Trigonometry and areas are blended in: tangent lengths, chord lengths, and shaded regions between tangents and arcs.
- Reasons must be exact. 'Z angles' or 'angles in a half circle' will not earn the C mark.

The mock exam format on your revision list is two 90 minute papers, one non-calculator and one calculator, covering all Year 10 and Year 11 work, so this topic should be secure from both angles: reasoning by hand, and sector and tangent calculations with a calculator.

# The Eight Theorems on One Page

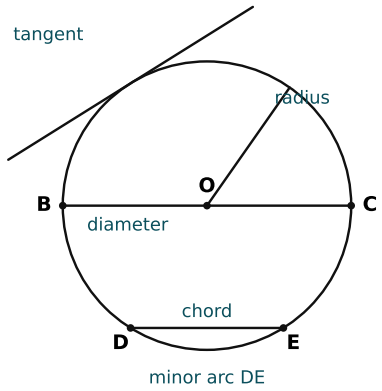
Learn the picture, the statement and the exact reason wording together. In the exam the picture is what you will recognise first.

| Picture | Theorem | Reason to write |
|---|---|---|
|   | **1. Angle at the centre**<br>Angle at the centre = 2 × angle at the circumference (same arc). | *Angle at the centre is twice the angle at the circumference.* |
|   | **2. Semicircle**<br>Angle in a semicircle = 90°. The chord must be a diameter. | *Angle in a semicircle is 90°.* |
|   | **3. Same segment**<br>Angles in the same segment are equal. | *Angles in the same segment are equal.* |
|   | **4. Cyclic quadrilateral**<br>Opposite angles add up to 180°. Exterior angle = interior opposite angle. | *Opposite angles of a cyclic quadrilateral add up to 180°.* |
|   | **5. Tangent and radius**<br>A tangent meets the radius at 90°. | *The angle between a tangent and a radius is 90°.* |
|   | **6. Two tangents**<br>Tangents from one external point are equal. OAPB is a kite. | *Tangents from an external point are equal in length.* |
|   | **7. Alternate segment**<br>Tangent-chord angle = angle in the alternate segment. | *Alternate segment theorem (angle between tangent and chord equals angle in the alternate segment).* |
|   | **8. Chords**<br>Perpendicular from the centre bisects the chord. | *The perpendicular from the centre to a chord bisects the chord.* |

# 1. Parts of a Circle, Circumference, Area and Sectors

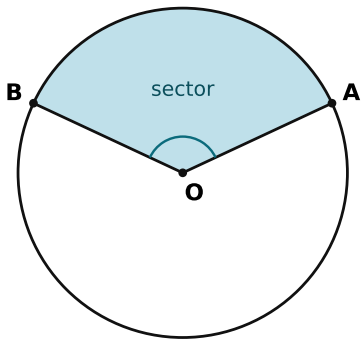
*Objectives: review the names of the parts of a circle (F). Review the area and circumference of circles and sectors (F).*

Every circle theorem question starts with vocabulary. If you cannot name a chord from a tangent or a sector from a segment, you cannot read the question, let alone answer it. This section is also where the exam hides easy marks and, at grade 9, where it hides the awkward combined problems: a sector, a triangle and a segment in the same diagram.



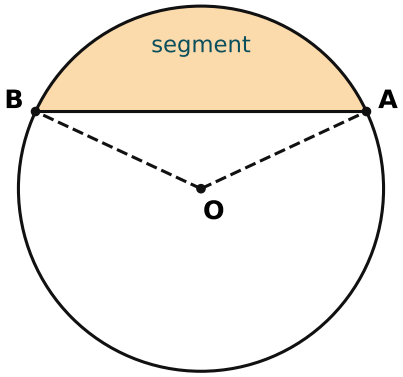

Parts of a circle

| Term | Meaning |
|---|---|
| Centre | The fixed point (O) in the middle. Every point on the circle is the same distance from it. |
| Radius | A line from the centre to the circumference. All radii of a circle are equal. |
| Diameter | A chord through the centre. Its length is twice the radius. |
| Chord | A straight line joining two points on the circumference. |
| Tangent | A straight line that touches the circle at exactly one point. |
| Arc | Part of the circumference. Shorter is the minor arc, longer is the major arc. |
| Circumference | The distance round the circle (also the name of the edge itself). |




Sector

- Sector: the region between two radii and the arc joining their ends. It looks like a slice of pie.
- Segment: the region between a chord and the arc it cuts off. It looks like a cap.
- Semicircle: half a circle, cut off by a diameter. It is both a sector and a segment.
- A chord splits a circle into two segments, a minor segment and a major segment.




Segment: the shaded cap between a chord and its arc

### The measures you must know

| Quantity | Formula | Notes |
|---|---|---|
| Circumference | $C = 2\pi r = \pi d$ | r is the radius, d the diameter |
| Area of a circle | $A = \pi r^2$ | Square the radius first, then multiply by π |
| Arc length | $\frac{\theta}{360} \times 2\pi r$ | θ is the angle at the centre in degrees |
| Area of a sector | $\frac{\theta}{360} \times \pi r^2$ | The same fraction of the area |
| Perimeter of a sector | arc length + 2r | Do not forget the two straight edges |
| Area of a segment | sector area − triangle area | Triangle area = $\frac{1}{2} r^2 \sin\theta$ |

**Why the sector formulas work.** A sector with angle θ is the fraction $\frac{\theta}{360}$ of the whole circle, so its arc is that fraction of the circumference and its area is that fraction of the circle's area. Nothing else is going on. If you remember that single idea you do not need to memorise two separate formulas.

**Exact answers.** If a question says 'in terms of π' leave π in your answer (for example 7π cm). If it says 'to 3 significant figures' use the π button on your calculator and round only at the end.

#### Example 1.1: Circumference and area

**A circle has radius 7.5 cm. Work out (a) the circumference and (b) the area. Give both answers in terms of π, then to 3 significant figures.**

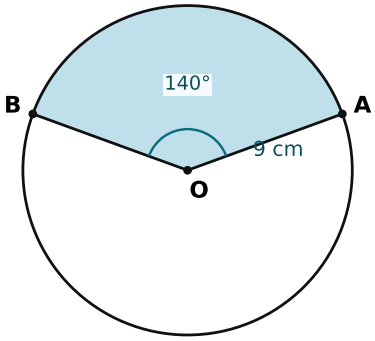
(a) $C = 2\pi r = 2 \times \pi \times 7.5 = 15\pi$ cm. Using a calculator, $15\pi = 47.12... = 47.1$ cm (3 s.f.).

(b) $A = \pi r^2 = \pi \times 7.5^2 = 56.25\pi$ cm². Using a calculator, $56.25\pi = 176.71... = 177$ cm² (3 s.f.).

**Answer: (a) 15π cm = 47.1 cm (b) 56.25π cm² = 177 cm²**

### Example 1.2: Arc length, sector area and perimeter

**A sector has radius 9 cm and angle 140°. Work out the arc length, the area and the perimeter of the sector. Give your answers to 3 significant figures.**




Fraction of the circle: $\frac{140}{360} = \frac{7}{18}$.

Arc length = $\frac{7}{18} \times 2\pi \times 9 = 7\pi = 21.99... = 22.0$ cm.

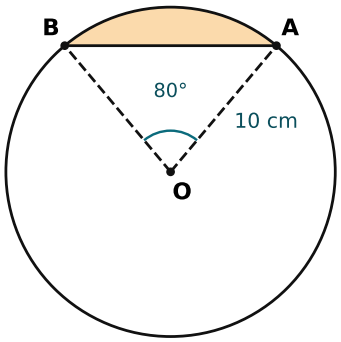
Area = $\frac{7}{18} \times \pi \times 9^2 = \frac{7}{18} \times 81\pi = 31.5\pi = 98.96... = 99.0$ cm².

Perimeter = arc + two radii = $7\pi + 9 + 9 = 39.99... = 40.0$ cm.

**Answer: Arc 22.0 cm, area 99.0 cm², perimeter 40.0 cm**

### Example 1.3: Area of a segment

**A chord AB of a circle of radius 10 cm subtends an angle of 80° at the centre O. Work out the area of the shaded segment. Give your answer to 3 significant figures.**




Sector OAB: $\frac{80}{360} \times \pi \times 10^2 = \frac{200\pi}{9} = 69.81...$ cm².

Triangle OAB: $\frac{1}{2} \times 10 \times 10 \times \sin 80° = 50 \sin 80° = 49.24...$ cm². (Two radii and the angle between them: use $\frac{1}{2}ab \sin C$.)

Segment = sector − triangle = $69.81... - 49.24... = 20.57...$ cm².

**Answer: 20.6 cm²**

### Example 1.4: Working backwards from an arc length

**A sector has angle 120° and arc length 8π cm. Work out the radius, then the area of the sector in terms of π.**

Arc length = $\frac{120}{360} \times 2\pi r = \frac{2\pi r}{3}$. So $\frac{2\pi r}{3} = 8\pi$.

Divide both sides by π and multiply by 3: $2r = 24$, so $r = 12$ cm.

Area = $\frac{1}{3} \times \pi \times 12^2 = \frac{1}{3} \times 144\pi = 48\pi$ cm².

**Answer: r = 12 cm, area = 48π cm²**

## Common mistakes

- Using the diameter where the radius is needed (or the reverse). Check which one the question gives.
- Forgetting to square the radius in area formulas.
- Forgetting that the perimeter of a sector includes both radii.
- Rounding π to 3.14 in a question that wants an exact answer or 3 s.f. Use the calculator π.
- Subtracting the wrong way for a segment: it is sector minus triangle, never triangle minus sector.

# 2. The Angle Toolkit You Need Before the Theorems

*Every circle theorem problem is solved by chaining basic angle facts with the circle theorems. Know these cold.*

### Basic angle facts

- Angles in a triangle add up to 180°.
- Angles on a straight line add up to 180°. Angles round a point add up to 360°.
- Angles in a quadrilateral add up to 360°.
- In an isosceles triangle the two base angles are equal.
- The exterior angle of a triangle equals the sum of the two interior opposite angles.
- Parallel lines: alternate angles are equal, corresponding angles are equal, co-interior angles add up to 180°.

**The most important single fact in this topic:** every radius of a circle has the same length. So any triangle formed by two radii is isosceles. A circle theorem diagram with a centre O and two radii drawn almost always contains a hidden isosceles triangle, and finding it is often the first step of a grade 8 or 9 solution.

### Example 2.1: A hidden isosceles triangle

**A and B lie on a circle with centre O. Angle AOB is 124°. Work out angle OAB.**

OA and OB are both radii, so triangle OAB is isosceles with OA = OB.

The two base angles are equal: angle OAB = angle OBA.

Angle OAB = (180° − 124°) ÷ 2 = 56° ÷ 2 = 28°.

**Answer: Angle OAB = 28°**

### How to write reasons (this is where marks are lost)

- In Edexcel mark schemes, a correct reason is worth its own communication (C) mark. A correct angle with no reason may score only the method mark.
- Name the theorem or fact in words, using the standard wording on the reason bank page of this workbook.
- Write the reason next to the line where you use it, not in a block at the end.
- Never write 'because it is a circle theorem' or 'Z angles'. Name the exact theorem.
- When you chain several steps, give a reason for every step. Marks are awarded per step.

### The five-step method for any circle problem

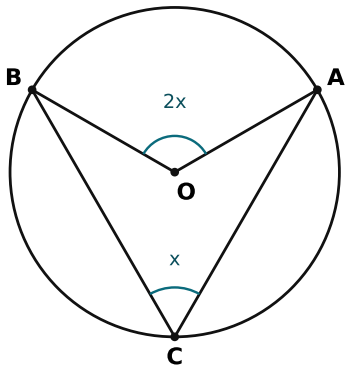
- 1. Mark everything you know on the diagram: equal radii, right angles at tangents, given angles.
- 2. Look for isosceles triangles made from radii.
- 3. Look for the theorem patterns: centre, semicircle, same segment, cyclic quadrilateral, tangent, alternate segment.
- 4. Work one angle at a time and write a reason for each.
- 5. Check the answer: do the angles in every triangle and quadrilateral still add up, and is the size sensible against the diagram?

# 3. Theorem 1: Angle at the Centre

*Objectives: recognise and apply circle theorems (H). Prove the theorem (grade 8 to 9).*

**The theorem**

- The angle at the centre of a circle is twice the angle at the circumference, when both angles are subtended by the same arc.



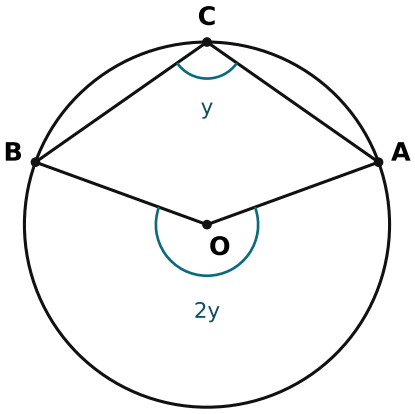

Angle AOB = 2 × angle ACB

- **Same arc** means both angles start from the same two points A and B on the circle.
- One angle has its vertex at the centre O. The other has its vertex C on the circumference.
- The centre angle opens twice as wide as the circumference angle.
- It does not matter where C is on the major arc: the angle at C stays the same.

**How to spot it**

- The centre O is marked and two lines from it go to points on the circle.
- You are given an angle at the centre and asked for one at the circumference, or the other way round.
- An angle at the circumference looks 'at' the same two points as an angle at the centre.

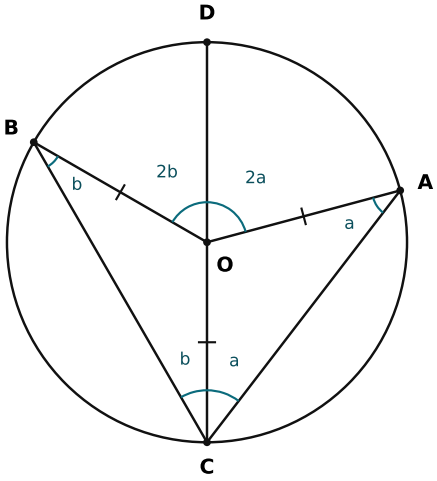
**The reflex case.** If C lies on the minor arc AB, the angle at C is obtuse and the angle at the centre that goes with it is the reflex angle AOB (bigger than 180°). The rule is unchanged: the reflex angle is still twice the angle at C. Many students forget this version, and it is a favourite grade 7 to 9 twist.




C on the minor arc: the reflex angle at O is twice the angle at C

**Proof of Theorem 1 (case where O lies inside angle ACB)**

1. Draw the line from C through O to meet the circle at D. Mark OA = OB = OC (radii). Let angle OCA = a and angle OCB = b.
2. Triangle OAC is isosceles (OA = OC, radii), so angle OAC = a [ *base angles of an isosceles triangle are equal* ].
3. Angle AOD is an exterior angle of triangle OAC, so angle AOD = a + a = 2a [ *exterior angle of a triangle equals the sum of the two interior opposite angles* ].
4. In the same way triangle OBC is isosceles, angle OBC = b, and angle BOD = 2b.
5. Angle ACB = a + b and angle AOB = 2a + 2b = 2(a + b).
6. So angle AOB = 2 × angle ACB. The cases where O is outside the angle, or on a side, follow in the same way using subtraction.




Proof diagram: the line CD through the centre splits the problem into two isosceles triangles

Write: **Angle at the centre is twice the angle at the circumference.**

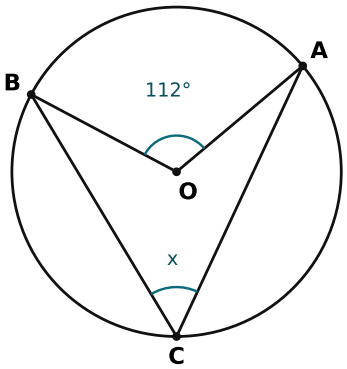
**Example 3.1: Direct use**

**A, B and C are points on a circle with centre O. Angle AOB = 112°. Work out angle ACB. Give a reason.**

Angle ACB = 112° ÷ 2 = 56°.

[ *The angle at the centre is twice the angle at the circumference.* ]

**Answer: Angle ACB = 56°**



### Example 3.2: The reflex case

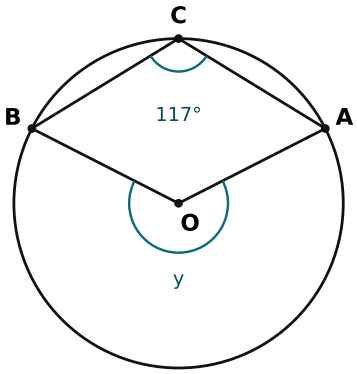
**A, B and C are points on a circle with centre O. C is on the minor arc AB and angle ACB = 117°. Work out the reflex angle AOB, and then the angle AOB that is less than 180°.**

Reflex angle AOB = 2 × 117° = 234°. [ *The angle at the centre is twice the angle at the circumference.* ]

The angle AOB that is less than 180° = 360° − 234° = 126°. [ *Angles round a point add up to 360°.* ]

Check: 126° is the size of the minor arc AB, and C sits on that arc, so an obtuse angle at C is expected.

**Answer: Reflex angle AOB = 234°, angle AOB = 126°**




### Example 3.3: Isosceles triangle first, then the theorem

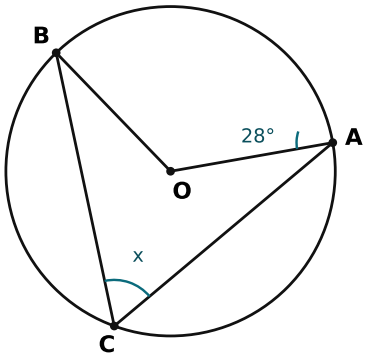
**A, B and C are on a circle with centre O. Angle OAB = 28°. Work out angle ACB.**

OA = OB (radii), so triangle OAB is isosceles and angle OBA = 28°. [ *Base angles of an isosceles triangle are equal.* ]

Angle AOB = 180° − 28° − 28° = 124°. [ *Angles in a triangle add up to 180°.* ]

Angle ACB = 124° ÷ 2 = 62°. [ *The angle at the centre is twice the angle at the circumference.* ]

**Answer: Angle ACB = 62°**




## Common mistakes

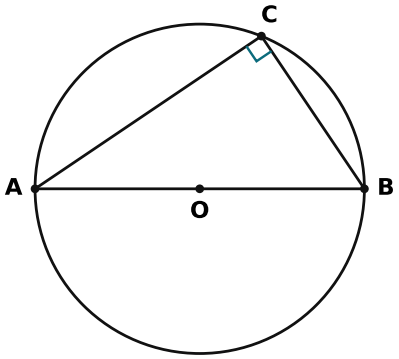
- Using the theorem when O is not the centre. If the centre is not marked or stated, you cannot use it.
- Applying it to angles that are not on the same arc, for example angles with different end points.
- Halving when you should double (or the reverse). The centre angle is always the bigger one.
- Forgetting the reflex version when C is on the minor arc.

# 4. Theorem 2: Angle in a Semicircle

*A special case of Theorem 1, and one of the most frequently tested facts.*

**The theorem**

- The angle in a semicircle is a right angle (90°).
- **Converse:** if the angle at the circumference between two chords is 90°, then the chord joining their ends is a diameter.




Angle ACB = 90°

- AB must be a diameter, so the line AB passes through the centre O.
- C can be anywhere else on the circumference. The angle at C is always 90°.
- Why: the angle at the centre on a straight line AB is 180°, and the angle at the circumference is half of that (Theorem 1).

**How to spot it**

- A diameter is drawn, or the question says 'AB is a diameter'.
- A triangle is drawn with one side as the diameter and the opposite vertex on the circle.
- A right angle is given at a point on the circle and you are asked to show that a line is a diameter (converse).

**Proof that the angle in a semicircle is 90°**

1. Join C to the centre O. OA = OB = OC (radii), so triangles OAC and OBC are isosceles.
2. Let angle OAC = a. Then angle OCA = a [ *base angles of an isosceles triangle* ]. Let angle OBC = b. Then angle OCB = b.
3. Angles in triangle ABC: a + b + (a + b) = 180° [ *angles in a triangle add up to 180°* ].
4. So 2(a + b) = 180°, so a + b = 90°. But angle ACB = a + b. So angle ACB = 90°.

Write: **Angle in a semicircle is 90°.** For the converse: **Angle in a semicircle is 90°, so AC is a diameter** (state the converse in words).

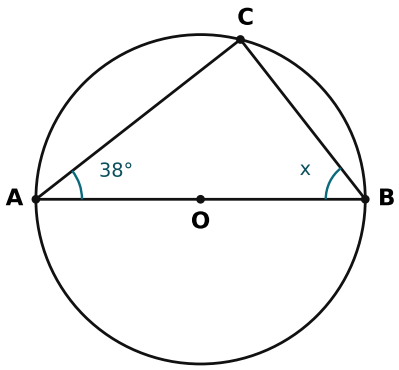
### Example 4.1: Finding the third angle

**AB is a diameter of a circle with centre O. C is a point on the circle. Angle CAB = 38°. Work out angle ABC.**

Angle ACB = 90°. [ *Angle in a semicircle is 90°.* ]

Angle ABC = 180° − 90° − 38° = 52°. [ *Angles in a triangle add up to 180°.* ]

**Answer: Angle ABC = 52°**




### Example 4.2: With algebra

**AB is a diameter. Angle BAC = (2x + 10)° and angle ABC = 3x°. Work out the value of x and the size of each angle.**

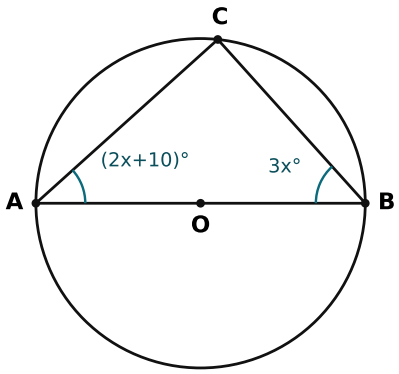
Angle ACB = 90° [ *angle in a semicircle is 90°* ].

So (2x + 10) + 3x + 90 = 180 [ *angles in a triangle add up to 180°* ].

5x + 100 = 180, so 5x = 80 and x = 16.

Angle BAC = 2 × 16 + 10 = 42° and angle ABC = 3 × 16 = 48°. Check: 42 + 48 + 90 = 180. ✓

**Answer: x = 16, angle BAC = 42°, angle ABC = 48°**




## Common mistakes

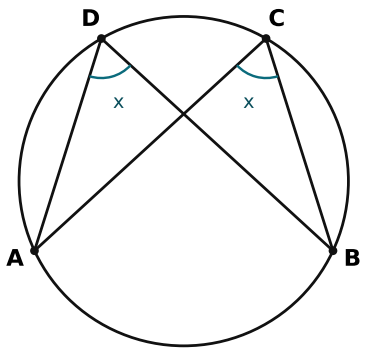
- Using the theorem when AB is not a diameter. Check the line really goes through O.
- Forgetting the right angle is at the point C on the circle, not at the centre.
- Not stating the converse in words when a question asks you to show that a line is a diameter.

# 5. Theorem 3: Angles in the Same Segment

*Also written as: angles subtended by the same arc at the circumference are equal.*

**The theorem**

- Angles in the same segment are equal. That is, angles at the circumference subtended by the same chord (or arc), on the same side of it, are equal.




Angle ACB = angle ADB

- C and D both lie on the same side of the chord AB.
- Both angles 'look at' the chord AB, so both equal half the centre angle AOB (Theorem 1).
- Hence they equal each other, without needing the centre at all.

**How to spot it**

- Two triangles share a chord as a common side, with their other vertices on the circle on the same side.
- A quadrilateral with both diagonals drawn: each pair of angles looking at the same side is equal. The diagram looks like a bow tie or a butterfly.

Write: **Angles in the same segment are equal.**

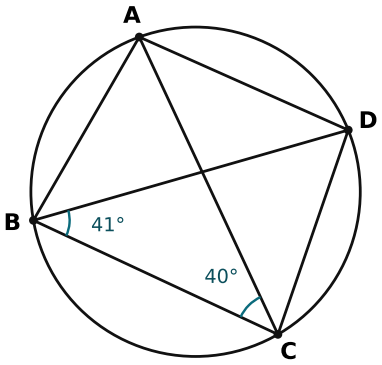
**Example 5.1: Two angles from a diagonal diagram**

**A, B, C and D are points on a circle. Angle DBC = 41° and angle ACB = 40°. Work out (a) angle DAC and (b) angle ADB.**

(a) Angles DAC and DBC are both subtended by the chord DC, from the same side. So angle DAC = 41°. [ *Angles in the same segment are equal.* ]

(b) Angles ADB and ACB are both subtended by the chord AB, from the same side. So angle ADB = 40°. [ *Angles in the same segment are equal.* ]

**Answer: (a) 41° (b) 40°**



### Example 5.2: With algebra

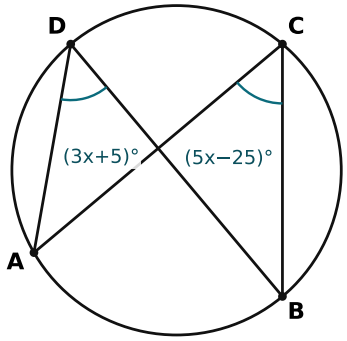
**Angle ADB = (3x + 5)° and angle ACB = (5x − 25)°. Work out the value of x and the size of angle ADB.**

Both angles are subtended by the chord AB from the same side, so they are equal. [ *Angles in the same segment are equal.* ]

3x + 5 = 5x − 25, so 30 = 2x and x = 15.

Angle ADB = 3 × 15 + 5 = 50°. Check: 5 × 15 − 25 = 50°. ✓

**Answer: x = 15, angle ADB = 50°**




## Common mistakes

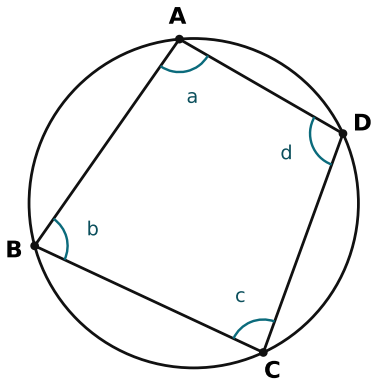
- Using angles on opposite sides of the chord. Those are not equal (they add up to 180° if the four points form a cyclic quadrilateral).
- Confusing 'same segment' with 'alternate segment'. The first is angle to angle, the second involves a tangent.

# 6. Theorem 4: Cyclic Quadrilaterals

*A cyclic quadrilateral has all four vertices on the circumference.*

**The theorem**

- Opposite angles of a cyclic quadrilateral add up to 180°.
- **Consequence:** the exterior angle of a cyclic quadrilateral equals the interior opposite angle.

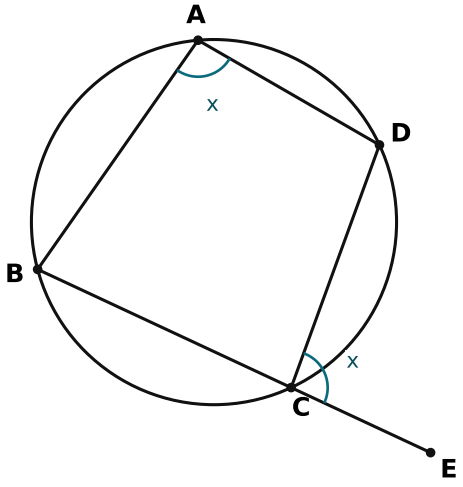



a + b = 180° and c + d = 180°

- Opposite angles are the ones that do not share a side, for example A and C, or B and D.
- Adjacent angles are not linked by this rule.
- The diagram does not need the centre to be shown.

**Proof that opposite angles add up to 180°**

1. Let O be the centre. Let angle ABC = x, with B on the major arc AC.
2. The angle at the centre AOC (on the same arc, less than 180°) = 2x [ *angle at the centre is twice the angle at the circumference* ].
3. The reflex angle AOC = 360° − 2x [ *angles round a point add up to 360°* ].
4. Angle ADC is at the circumference and is subtended by the same arc as the reflex angle. So angle ADC = (360° − 2x) ÷ 2 = 180° − x [ *angle at the centre is twice the angle at the circumference* ].
5. So angle ABC + angle ADC = x + (180° − x) = 180°.




Exterior angle: angle DCE = angle DAB

**Why the exterior angle fact holds.** Angle DCE and angle BCD lie on a straight line, so angle DCE = 180° − angle BCD. But angle BCD = 180° − angle DAB (opposite angles). So angle DCE = angle DAB.

Write: **Opposite angles of a cyclic quadrilateral add up to 180°.** For the exterior angle: **The exterior angle of a cyclic quadrilateral equals the interior opposite angle.**

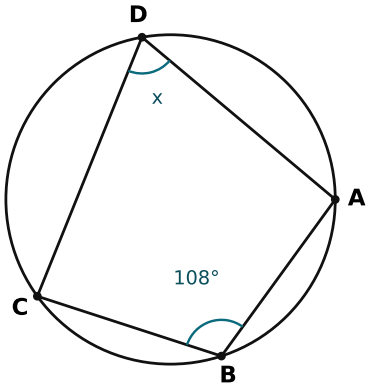
### Example 6.1: Direct use

**ABCD is a cyclic quadrilateral. Angle ABC = 108°. Work out angle ADC. Give a reason.**

Angle ADC = 180° − 108° = 72°.

[ *Opposite angles of a cyclic quadrilateral add up to 180°.* ]

**Answer: Angle ADC = 72°**




### Example 6.2: With algebra

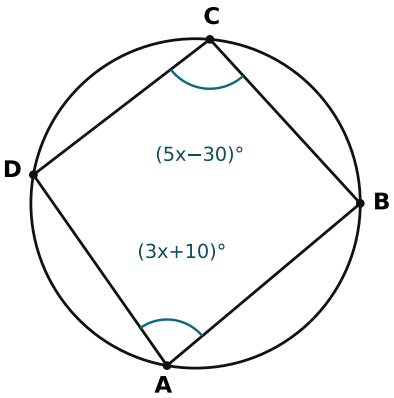
**ABCD is a cyclic quadrilateral. Angle DAB = (3x + 10)° and angle BCD = (5x − 30)°. Work out the value of x and the size of both angles.**

Angles DAB and BCD are opposite, so (3x + 10) + (5x − 30) = 180 [ *opposite angles of a cyclic quadrilateral add up to 180°* ].

8x − 20 = 180, so 8x = 200 and x = 25.

Angle DAB = 3 × 25 + 10 = 85° and angle BCD = 5 × 25 − 30 = 95°. Check: 85 + 95 = 180. ✓

**Answer: x = 25, angle DAB = 85°, angle BCD = 95°**




### Example 6.3: Exterior angle

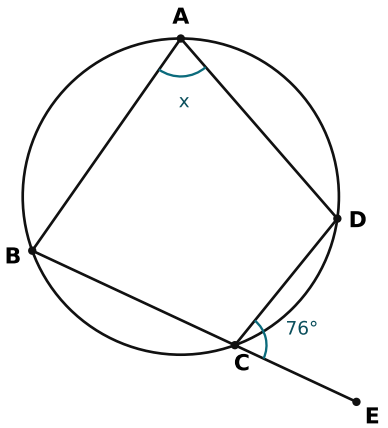
**ABCD is a cyclic quadrilateral. The side BC is extended to E. Angle DCE = 76°. Work out angle DAB.**

Angle DAB = angle DCE = 76°.

[ *The exterior angle of a cyclic quadrilateral equals the interior opposite angle.* ]

Alternatively: angle BCD = 180° − 76° = 104° (angles on a straight line), then angle DAB = 180° − 104° = 76° (opposite angles). Both routes give the same answer.

**Answer: Angle DAB = 76°**



## Common mistakes

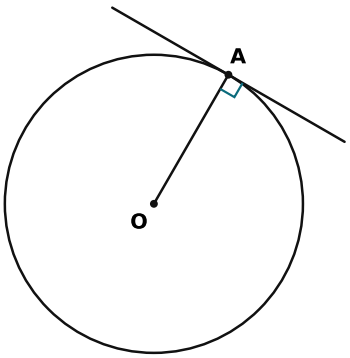
- Only use this theorem if all four vertices are on the circle.
- Adding adjacent angles to 180°. It is opposite angles that add up to 180°.
- Taking the exterior angle to equal the adjacent interior angle. It equals the opposite one.

# 7. Theorem 5: Tangent and Radius

*Tangents bring right angles into the diagram, which means Pythagoras and trigonometry are often needed.*

**The theorem**

- A tangent to a circle is perpendicular to the radius drawn to the point of contact.




The angle between a tangent and the radius at the point of contact is 90°

- Draw the radius from O to the point where the tangent touches the circle. You get an angle of 90°.
- Why: every other point on the tangent is outside the circle, so the point of contact is the closest point of the line to O. The shortest distance from a point to a line is the perpendicular.
- The right angle gives you right-angled triangles, so use Pythagoras and sin, cos, tan.

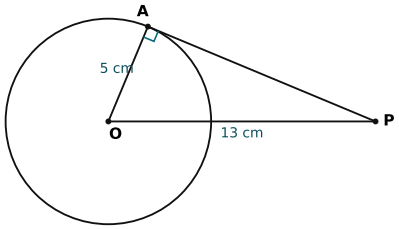
**How to spot it**

- The word 'tangent', or a line that touches the circle at one point.
- A radius or the centre O drawn to the point of contact.
- Lengths are given and a missing length is wanted: that means Pythagoras. Angles and one length: trigonometry.

Write: **The angle between a tangent and the radius is 90°.**

**Example 7.1: Tangent length with Pythagoras and trigonometry**

**PA is a tangent to a circle with centre O at the point A. The radius is 5 cm and OP = 13 cm. (a) Work out the length PA. (b) Work out angle OPA to 1 decimal place.**




Angle OAP = 90°. [ *The angle between a tangent and the radius is 90°.* ]

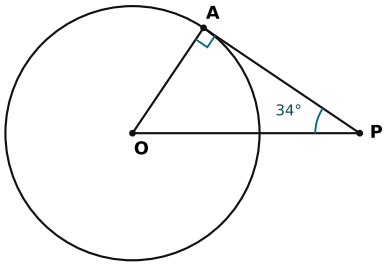
(a) $PA^2 = OP^2 - OA^2 = 13^2 - 5^2 = 169 - 25 = 144$, so PA = 12 cm. [ *Pythagoras' theorem.* ]

(b) $\sin(OPA) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{OA}{OP} = \frac{5}{13}$. So angle OPA = $\sin^{-1}(\frac{5}{13}) = 22.6°$ (1 d.p.).

**Answer: (a) PA = 12 cm (b) angle OPA = 22.6°**

### Example 7.2: Angle chase with a tangent

**PA is a tangent to a circle with centre O at A. Angle OPA = 34°. Work out angle POA.**




Angle OAP = 90°. [ *The angle between a tangent and the radius is 90°.* ]

Angle POA = 180° − 90° − 34° = 56°. [ *Angles in a triangle add up to 180°.* ]

**Answer: Angle POA = 56°**

## Common mistakes

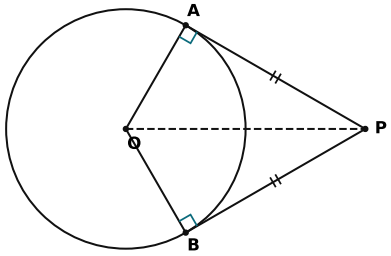
- Forgetting to mark the right angle as soon as you see a tangent.
- Using the tangent length as if it were a radius, or treating OP as the tangent.
- Rounding too early in trigonometry. Keep the full calculator value until the end.

# 8. Theorem 6: Two Tangents from a Point

*Tangents drawn from the same external point are equal in length, and the diagram is a kite.*

### The theorem

- Tangents drawn to a circle from the same external point are equal in length.
- **Kite properties:** OAPB is a kite with two right angles at A and B. OP bisects angle APB, bisects angle AOB, and is the perpendicular bisector of the chord AB.




PA = PB

- Triangle APB is isosceles (PA = PB), so angle PAB = angle PBA.
- Angles in the kite OAPB add up to 360°, with right angles at A and B.
- So angle AOB + angle APB = 180°.

### Proof that PA = PB

1. Join OA, OB and OP. Compare triangles OAP and OBP.
2. OA = OB [ *radii of the same circle* ].
3. Angle OAP = angle OBP = 90° [ *a tangent is perpendicular to the radius* ].
4. OP is common to both triangles.
5. So the triangles are congruent (RHS: right angle, hypotenuse, side). Therefore PA = PB.

Write: **Tangents from an external point are equal in length.**

### Example 8.1: Angles in the tangent diagram

**PA and PB are tangents to a circle with centre O. Angle APB = 50°. Work out (a) angle PAB and (b) angle AOB.**

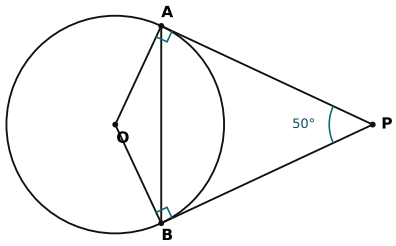
(a) PA = PB [ *tangents from an external point are equal in length* ], so triangle APB is isosceles.

Angle PAB = (180° − 50°) ÷ 2 = 65°. [ *Base angles of an isosceles triangle are equal; angles in a triangle add up to 180°.* ]

(b) Angles OAP and OBP are 90° [ *tangent perpendicular to radius* ].

Angle AOB = 360° − 90° − 90° − 50° = 130°. [ *Angles in a quadrilateral add up to 360°.* ]

Check: angle OAB = 90° − 65° = 25°, and in isosceles triangle OAB, (180° − 130°) ÷ 2 = 25°. ✓




**Answer: (a) 65° (b) 130°**

### Example 8.2: Equal tangents with algebra

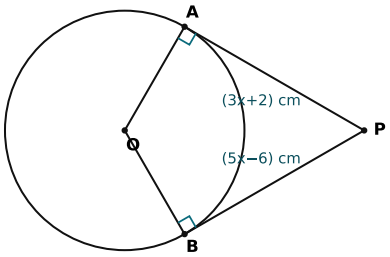
**PA and PB are tangents to a circle. PA = (3x + 2) cm and PB = (5x − 6) cm. Work out the length of PA.**

PA = PB [ *tangents from an external point are equal in length* ].

3x + 2 = 5x − 6, so 8 = 2x and x = 4.

PA = 3 × 4 + 2 = 14 cm. Check: PB = 5 × 4 − 6 = 14 cm. ✓

**Answer: PA = 14 cm**




## Common mistakes

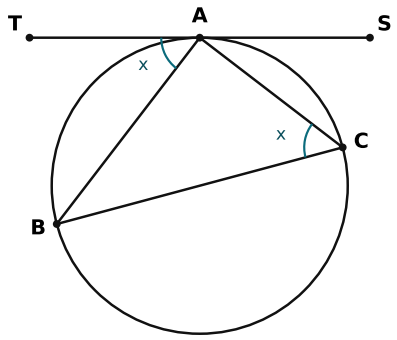
- Forgetting that the right angles at A and B are there. The kite angle sum depends on them.
- Treating OP as a tangent. It is a line of symmetry, not a tangent.
- Not writing the equal-tangents reason before using the isosceles triangle.

# 9. Theorem 7: The Alternate Segment Theorem

*The one that students find hardest to spot. Learn the picture, not just the words.*

### The theorem

- The angle between a tangent and a chord at the point of contact is equal to the angle in the alternate segment.




Angle TAB = angle ACB

- The **chord** is AB. The tangent TS touches the circle at A, one end of the chord.
- The **alternate segment** is the segment on the opposite side of the chord from the angle you are looking at.
- So the angle between the tangent and the chord on one side equals the angle at C, where C is on the other side of AB.

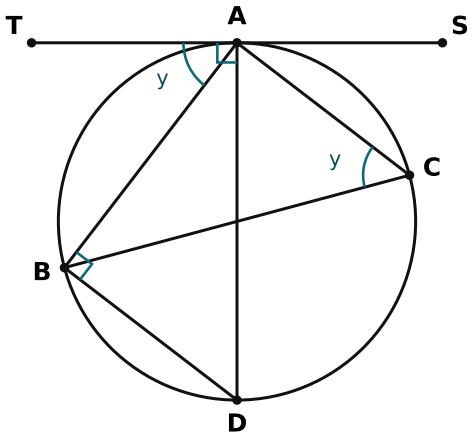
### How to spot it

- A tangent is drawn and the point of contact is also a vertex of a triangle inscribed in the circle.
- The chord starts at the point of contact.
- You are told an angle between the tangent and a chord, or asked for an angle at the circumference in the 'opposite' segment.

**How to avoid choosing the wrong angle.** Put your pencil on the angle between the tangent and the chord. Follow the chord to its other end. The angle you want is the one at the circumference that looks back at that chord from the other side of it, not from the same side as the tangent angle.

### Proof of the alternate segment theorem

1. Let TS be the tangent at A, with chord AB and C in the alternate segment. Let angle TAB = y. Draw the diameter AD and join BD.
2. Angle TAD = 90° [ *a tangent is perpendicular to the radius, and a diameter contains a radius* ]. So angle DAB = 90° − y.
3. Angle ABD = 90° [ *angle in a semicircle is 90°* ].
4. In triangle ABD: angle ADB = 180° − 90° − (90° − y) = y [ *angles in a triangle add up to 180°* ].
5. Angle ACB = angle ADB = y [ *angles in the same segment are equal* ].
6. So angle TAB = y = angle ACB. ∎




Proof diagram: add the diameter AD and join BD

Write: **Angle between tangent and chord equals the angle in the alternate segment** (Edexcel mark schemes accept 'alternate segment theorem').

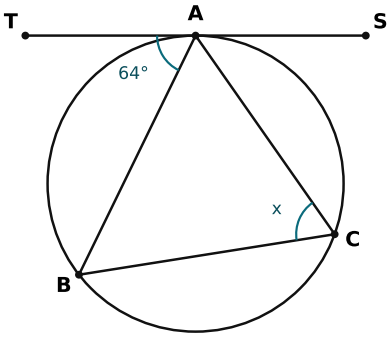
### Example 9.1: Direct use

**TAS is a tangent to the circle at A. Angle TAB = 64°. Work out angle ACB. Give a reason.**

Angle ACB = angle TAB = 64°.

[ *The angle between the tangent and the chord equals the angle in the alternate segment.* ]

**Answer: Angle ACB = 64°**




### Example 9.2: Both sides of the chord, then the triangle

**TAS is a tangent at A. Angle TAB = 52° and angle SAC = 71°. Work out the three angles of triangle ABC.**

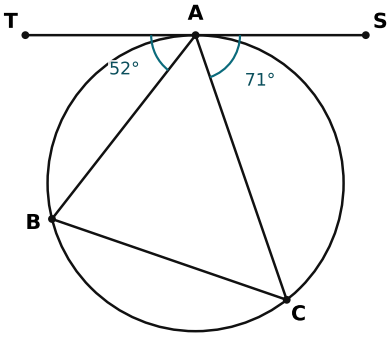
Angle ACB = angle TAB = 52°. [ *Alternate segment theorem.* ]

Angle ABC = angle SAC = 71°. [ *Alternate segment theorem, using the chord AC.* ]

Angle BAC = 180° − 52° − 71° = 57°. [ *Angles on a straight line add up to 180°.* ]

Check: 52° + 71° + 57° = 180°. ✓

**Answer: Angle ACB = 52°, angle ABC = 71°, angle BAC = 57°**



### Common mistakes

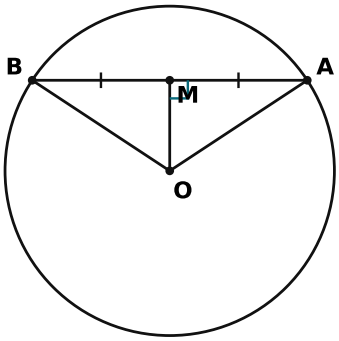
- Choosing the angle in the same segment instead of the alternate segment. That is the commonest error.
- Not stating 'alternate segment' as the reason. A named reason earns the C mark.
- Forgetting that each chord from the point of contact gives its own tangent-chord angle.

# 10. Theorem 8: Chords and the Centre

*Often combined with Pythagoras or trigonometry to find lengths and angles.*

**The theorem**

- The perpendicular from the centre of a circle to a chord bisects the chord.
- **Converse:** the line from the centre to the midpoint of a chord is perpendicular to the chord.
- Equal chords are the same distance from the centre.




OM is perpendicular to AB, so AM = MB

- Triangle OAB is isosceles (OA = OB, radii).
- The line from the apex of an isosceles triangle perpendicular to the base is also the median, so it bisects the base.
- Right-angled triangle OMA gives $\mathbf{r^2 = d^2 + (\text{half chord})^2}$, where d is the distance from the centre to the chord.

**How to spot it**

- A chord and a radius are given, and the 'distance from the centre' or the 'perpendicular distance' is mentioned.
- Two parallel chords of different lengths.
- A missing angle at the centre when a chord length and the radius are known.

Write: **The perpendicular from the centre to a chord bisects the chord.**

**Example 10.1: Distance from the centre**

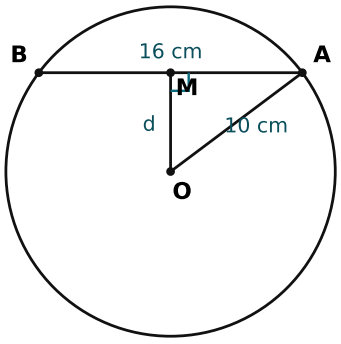
**A chord AB of length 16 cm is drawn in a circle of radius 10 cm, centre O. Work out the perpendicular distance from O to AB.**

Let M be the midpoint of AB. Then OM is perpendicular to AB and AM = 8 cm. [ *The perpendicular from the centre to a chord bisects the chord.* ]

In right-angled triangle OMA: $OM^2 = OA^2 - AM^2 = 10^2 - 8^2 = 100 - 64 = 36$. [ *Pythagoras' theorem.* ]

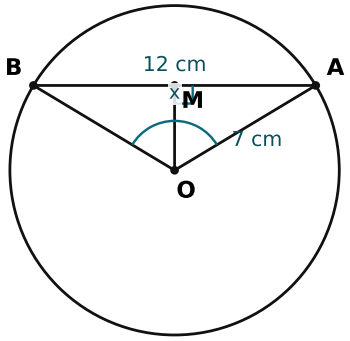
OM = 6 cm.

**Answer: 6 cm**



### Example 10.2: Angle at the centre from a chord

**A chord AB of length 12 cm is drawn in a circle of radius 7 cm, centre O. Work out angle AOB to 1 decimal place.**




Let M be the midpoint of AB. Then angle OMA = 90° and AM = 6 cm. [ *The perpendicular from the centre to a chord bisects the chord.* ]

In triangle OMA, $\sin(\text{AOM}) = \frac{\text{AM}}{\text{OA}} = \frac{6}{7}$. So angle AOM = $\sin^{-1}(\frac{6}{7}) = 58.99...°$.

Angle AOB = 2 × 58.99...° = 118.0°. (Triangle OAB is isosceles, so OM bisects angle AOB.)

**Answer: Angle AOB = 118.0°**

## Common mistakes

- Using the full chord length as one leg of the right-angled triangle. You need half of it.
- Forgetting that the 'distance' from the centre to a chord is always the perpendicular distance.
- Using the diameter by mistake when the radius is needed.

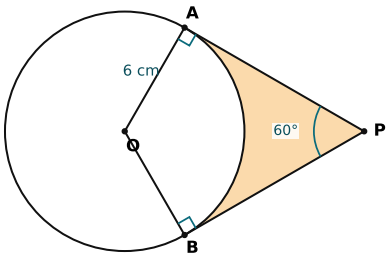
# 11. Combining Circle Theorems with Lengths and Areas (Grade 8 to 9)

*Top-grade questions blend theorems, trigonometry and sector areas into one problem.*

The usual structure is: (1) use a theorem to find an angle, (2) use that angle with trigonometry or Pythagoras to find a length, (3) use the length and angle to find an area, (4) subtract or add areas to get the shaded region. Plan the route before calculating. Mark each stage clearly so that method marks can be awarded even if a later slip happens.

### Example 11.1: Area between two tangents and the arc

**PA and PB are tangents to a circle with centre O and radius 6 cm. Angle APB = 60°. Work out the area of the shaded region, bounded by PA, PB and the minor arc AB. Give your answer to 3 significant figures.**




Angles OAP and OBP are 90° [ *tangent perpendicular to radius* ].

Angle AOB = 360° − 90° − 90° − 60° = 120° [ *angles in a quadrilateral add up to 360°* ]. So angle AOP = 60°.

In right-angled triangle OAP: $\tan 60° = \frac{AP}{OA}$, so AP = 6 tan 60° = 6√3 = 10.39... cm.

Area of kite OAPB = $2 \times \frac{1}{2} \times 6 \times 6\sqrt{3} = 36\sqrt{3} = 62.35...$ cm².

Area of sector OAB = $\frac{120}{360} \times \pi \times 6^2 = 12\pi = 37.70...$ cm².

Shaded area = kite − sector = 62.35... − 37.70... = 24.65... = 24.7 cm².

**Answer: 24.7 cm²**

# 12. Writing Circle Theorem Proofs (Grade 8 to 9)

*Edexcel Higher papers regularly include a 'Prove that' or 'Show that' question on a circle theorem.*

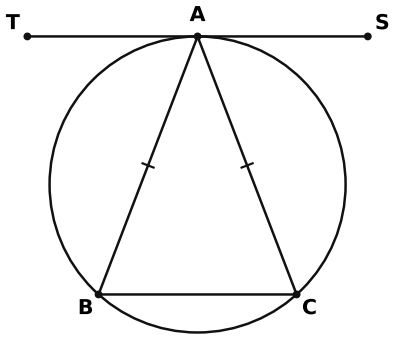
A proof is a chain of statements, each one justified. The examiner is not checking that the answer is true (they told you it is). They are checking that every step is valid and that nothing is assumed. Three habits earn the marks.

### Proof checklist

- 1. Define your letters. Say what each unknown angle is called ('Let angle OCA = a').
- 2. Put a reason after every statement. Use the standard theorem wording.
- 3. Finish by stating what you set out to prove, in the same words as the question.
- 4. Do not start from the answer and work backwards. Start from what is given.

### Example 12.1: Proof: the tangent is parallel to BC

**A, B and C are points on a circle. AB = AC. TAS is the tangent to the circle at A. Prove that TAS is parallel to BC.**




Let angle ABC = x. Since AB = AC, triangle ABC is isosceles, so angle ACB = x [ *base angles of an isosceles triangle are equal* ].

Angle TAB = angle ACB = x [ *the angle between a tangent and a chord equals the angle in the alternate segment* ].

So angle TAB = angle ABC = x.

These are alternate angles, and they are equal, so TAS is parallel to BC [ *alternate angles are equal when lines are parallel (converse)* ]. ∎

**Answer: Proved: TAS ∥ BC**

### Common mistakes

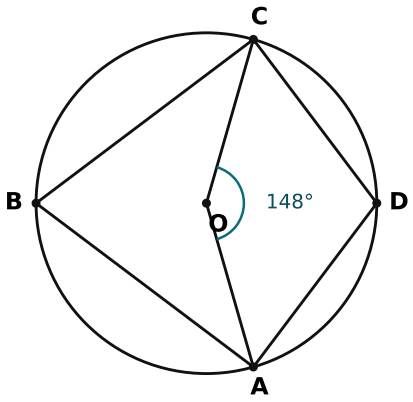
- Writing the answer first and 'proving' by plugging in the numbers. A numerical check is not a proof.
- Missing reasons. A correct chain with no reasons scores low.
- Using a theorem that has not been established, for example assuming the angle at the centre without naming it.
- Not defining the letters you use for angles.

# 13. Exam-Style Worked Questions

Original questions written in the style of Edexcel GCSE (9-1) Mathematics 1MA1 Higher, with full mark schemes. Cover the solution, attempt the question, then mark yourself. Codes: M = method, A = accuracy, B = independent mark, C = communication (reason).

**Exam-style 1 (Edexcel style, grade 6)** Non-calculator | 3 marks

A, B, C and D are points on a circle, centre O. Angle AOC = 148°. Work out the size of angle ADC. Give reasons for your answer.




**Mark scheme and worked solution**

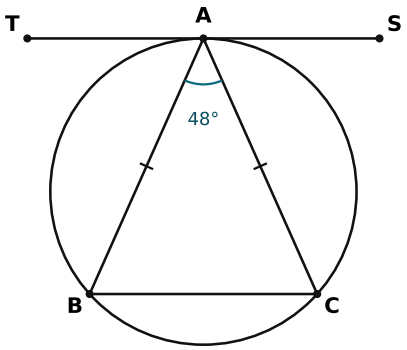
Angle ABC = 148° ÷ 2 = 74° [ *the angle at the centre is twice the angle at the circumference* ] **(M1)**

Angle ADC = 180° − 74° = 106° [ *opposite angles of a cyclic quadrilateral add up to 180°* ] **(A1)**

Both reasons stated **(C1)**

**Exam-style 2 (Edexcel style, grade 7)** Non-calculator | 4 marks

A, B and C are points on a circle. AB = AC and angle BAC = 48°. TAS is the tangent to the circle at A. Work out the size of angle TAB. Give reasons for your answer.




**Mark scheme and worked solution**

Triangle ABC is isosceles, so angle ACB = (180° − 48°) ÷ 2 = 66° **(M1)**

Reason: base angles of an isosceles triangle are equal, angles in a triangle add up to 180° **(C1)**

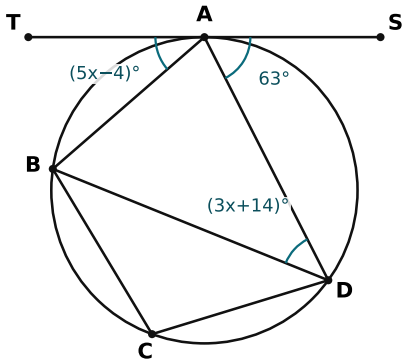
Angle TAB = angle ACB = 66° **(A1)**

Reason: the angle between the tangent and a chord is equal to the angle in the alternate segment **(C1)**

## Exam-style 3 (Edexcel style, grade 8)

Non-calculator | 5 marks

TAS is a tangent to the circle at A. Angle TAB = (5x − 4)°, angle ADB = (3x + 14)° and angle SAD = 63°.
(a) Work out the value of x. Give a reason for each step.(3)
(b) Work out the size of angle BCD.(2)



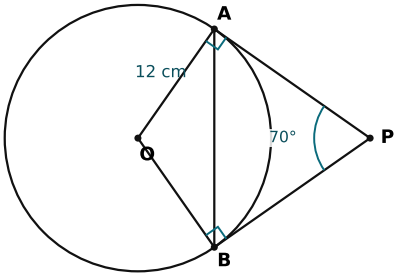

**Mark scheme and worked solution**

(a) Angle TAB = angle ADB [ *alternate segment theorem* ] **(C1)**
5x − 4 = 3x + 14, so 2x = 18 **(M1)**
x = 9 **(A1)**
(b) Angle TAB = 5 × 9 − 4 = 41°. Angle BAD = 180° − 41° − 63° = 76° [ *angles on a straight line add up to 180°* ] **(M1)**
Angle BCD = 180° − 76° = 104° [ *opposite angles of a cyclic quadrilateral add up to 180°* ] **(A1)**

## Exam-style 4 (Edexcel style, grade 9)

Calculator | 5 marks

PA and PB are tangents to a circle with centre O and radius 12 cm. Angle APB = 70°.
(a) Work out the size of angle AOP. Give reasons.(1)
(b) Work out the length of PA. Give your answer to 3 significant figures.(2)
(c) Work out the length of the chord AB. Give your answer to 3 significant figures.(2)




**Mark scheme and worked solution**

(a) Angle OAP = angle OBP = 90° [ *tangent perpendicular to radius* ]. Angle AOB = 360° − 90° − 90° − 70° = 110° [ *angles in a quadrilateral* ]. Angle AOP = 110° ÷ 2 = 55° [ *kite OAPB is symmetrical about OP* ] **(B1)**

(b) $\tan 55° = \frac{PA}{12}$ **(M1)** so PA = 12 tan 55° = 17.1 cm (3 s.f.) **(A1)**

(c) AB = 2 × 12 × sin 55° **(M1)** since triangle OAB is isosceles, so the perpendicular from O splits AB in half = 19.7 cm (3 s.f.) **(A1)**
Alternative for (c): cosine rule $AB^2 = 12^2 + 12^2 − 2 × 12 × 12 × \cos 110° = 288 − 288 \cos 110° = 386.5...$, so AB = 19.7 cm.

# Reason Bank

Copy these exactly. A correct angle with a weak or vague reason loses the C mark. The right-hand column shows the kind of wording that does not score.

| Theorem | Write this | Do not write this |
|---|---|---|
| **Angle at the centre** | Angle at the centre is twice the angle at the circumference | angle at the middle is double |
| **Semicircle** | Angle in a semicircle is 90° | angle in a half circle |
| **Same segment** | Angles in the same segment are equal | angles on the same side are equal |
| **Cyclic quadrilateral** | Opposite angles of a cyclic quadrilateral add up to 180° | angles in a quadrilateral are 180° |
| **Tangent and radius** | The angle between a tangent and a radius is 90° | right angle at the edge |
| **Two tangents** | Tangents from an external point are equal in length | the two lines are the same |
| **Alternate segment** | The angle between a tangent and a chord is equal to the angle in the alternate segment | Z angles, F angles, alternate angles (not in this context) |
| **Chord bisected** | The perpendicular from the centre to a chord bisects the chord | the line cuts it in half |
| **Isosceles from radii** | Triangle OAB is isosceles because OA and OB are radii | it looks isosceles |
| **Straight line, triangle, quadrilateral** | Angles on a straight line add up to 180°; angles in a triangle add up to 180°; angles in a quadrilateral add up to 360° | angle rule |

# Revision Sheet

Key facts on one page. Test yourself by covering the right side.

### Centre

Angle at centre = 2 × angle at circumference. Reflex angle goes with an obtuse angle on the minor arc.

### Semicircle

Diameter gives 90° at the circumference. Converse: 90° means the chord is a diameter.

### Same segment

Angles from the same chord on the same side are equal.

### Cyclic quadrilateral

Opposite angles sum to 180°. Exterior angle = interior opposite angle.

### Tangent

Tangent ⊥ radius. Tangents from a point are equal. OAPB is a kite: angle AOB + angle APB = 180°.

### Alternate segment

Tangent-chord angle = angle in the alternate segment (other side of the chord).

### Chords

Perpendicular from the centre bisects the chord. $r^2 = d^2 + (\text{half chord})^2$.

### Sectors

Arc = $\frac{\theta}{360} \times 2\pi r$. Sector = $\frac{\theta}{360} \times \pi r^2$. Segment = sector − $\frac{1}{2} r^2 \sin\theta$.

### Always

Look for isosceles triangles from two radii. Write a reason for every angle. Check angle sums at the end.

# Classwork

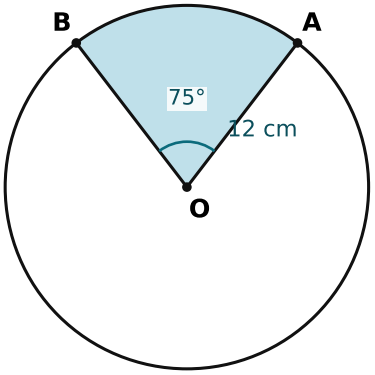
15 questions, 58 marks. Done with your tutor. Write a reason for every angle you find.

**CW1** Sectors — Grade 4 — Non-calculator — **[4 marks]**

The diagram shows a sector of a circle of radius 12 cm. The angle at the centre is 75°.

(a) Work out the arc length of the sector. Give your answer in terms of π. **(2)**

(b) Work out the area of the sector. Give your answer correct to 3 significant figures. **(2)**

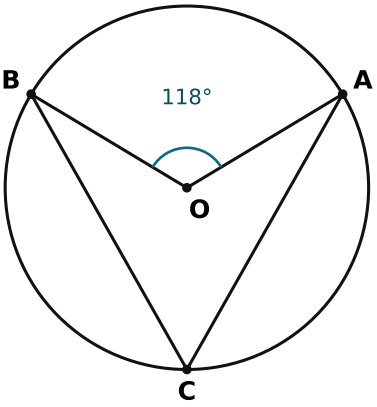



Working (give a reason for every angle)

Answer: _____

**CW2** Angle at the centre — Grade 4 — Non-calculator — **[2 marks]**

A, B and C are points on the circumference of a circle with centre O. Angle AOB = 118°. Work out the size of angle ACB. Give a reason for your answer. **(2)**



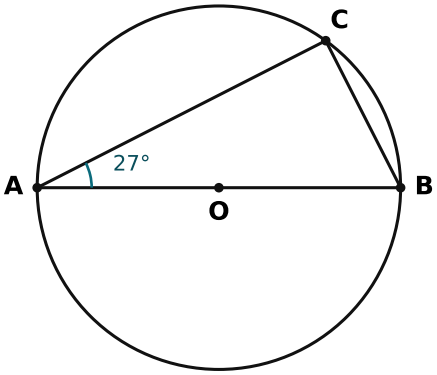

Working (give a reason for every angle)

Answer: _____

**CW3** Angle in a semicircle — Grade 4 — Non-calculator — **[2 marks]**

AB is a diameter of the circle. C is a point on the circumference. Angle CAB = 27°. Work out the size of angle ABC. Give reasons for your answer.

**(2)**




Working (give a reason for every angle)

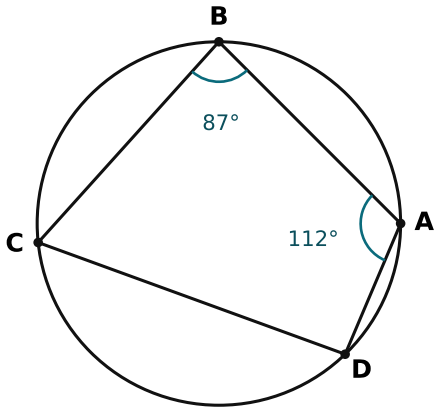
Answer: _____

**CW4** Cyclic quadrilateral — Grade 5 — Non-calculator — **[4 marks]**

A, B, C and D are points on a circle. Angle DAB = 112° and angle ABC = 87°.

(a) Work out the size of angle BCD. Give a reason. **(2)**

(b) Work out the size of angle ADC. Give a reason. **(2)**



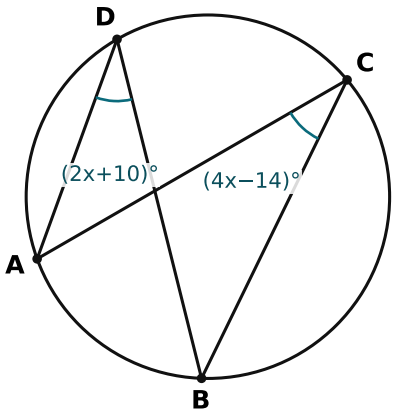

Working (give a reason for every angle)

Answer: _____

**CW5** Angles in the same segment — Grade 6 — Non-calculator — **[3 marks]**

A, B, C and D are points on a circle. Angle ADB = (2x + 10)° and angle ACB = (4x − 14)°. Work out the value of x and the size of angle ADB. Give a reason.

**(3)**



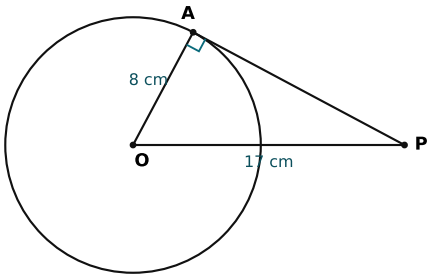

Working (give a reason for every angle)

Answer: _____

**CW6** Tangent and radius — Grade 6 — Non-calculator — **[3 marks]**

PA is a tangent to a circle with centre O. A is the point of contact. The radius of the circle is 8 cm and OP = 17 cm. Work out the length of PA.

**(3)**



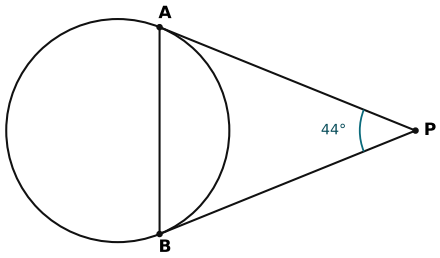

Working (give a reason for every angle)

Answer: _____

**CW7** Two tangents — Grade 6 — Non-calculator — **[3 marks]**

PA and PB are tangents to a circle. Angle APB = 44°. Work out the size of angle PAB. Give reasons for your answer.

**(3)**




Working (give a reason for every angle)

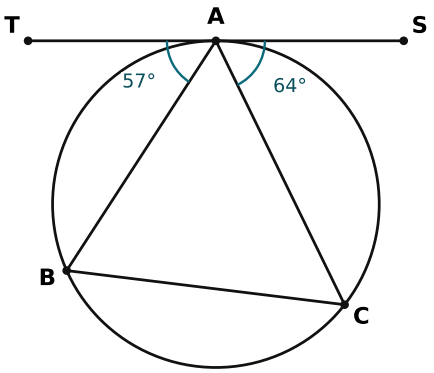
Answer: _____

**CW8** Alternate segment theorem — Grade 7 — Non-calculator — **[4 marks]**

TAS is a tangent to the circle at A. Angle TAB = 57° and angle SAC = 64°.

(a) Work out the size of angle ACB. Give a reason. **(2)**

(b) Work out the size of angle BAC. **(2)**



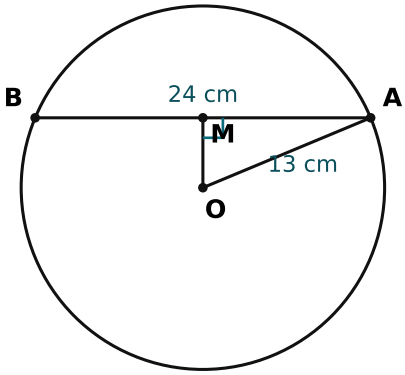

Working (give a reason for every angle)

Answer: _____

**CW9** Chords — Grade 7 — Non-calculator — **[3 marks]**

AB is a chord of a circle with centre O and radius 13 cm. AB = 24 cm. M is the midpoint of AB. Work out the distance OM.

**(3)**




Working (give a reason for every angle)

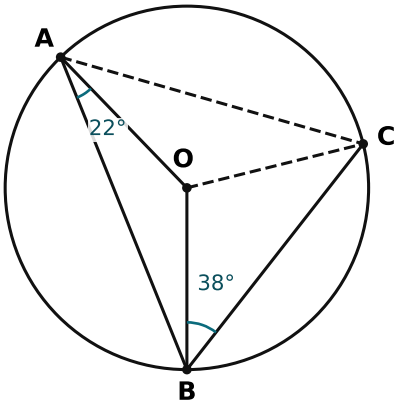
Answer: _____

**CW10** Centre theorem and isosceles triangles — Grade 7 — Non-calculator — **[4 marks]**

A, B and C are points on a circle with centre O. Angle OAB = 22° and angle OBC = 38°.

(a) Work out the size of angle ABC. Give reasons. **(2)**

(b) Work out the size of angle AOC. Give a reason. **(2)**




Working (give a reason for every angle)

Answer: _____

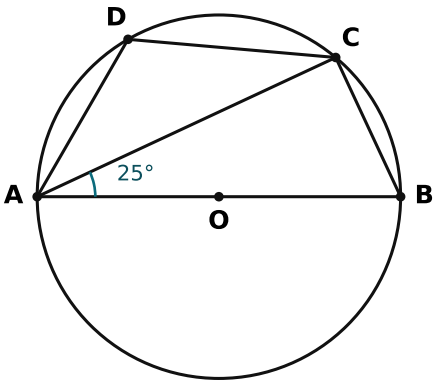
**CW11** Semicircle and cyclic quadrilateral — Grade 7 — Non-calculator — **[4 marks]**

AB is a diameter of the circle with centre O. A, B, C and D are points on the circle. Angle CAB = 25°.

(a) Write down the size of angle ACB. Give a reason. **(1)**

(b) Work out the size of angle ABC. **(1)**

(c) Work out the size of angle ADC. Give a reason. **(2)**




Working (give a reason for every angle)

Answer: _____

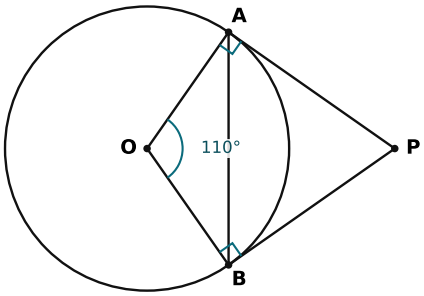
**CW12** Tangents and the kite — Grade 8 — Non-calculator — **[5 marks]**

A and B are points on a circle with centre O. PA and PB are tangents to the circle. Angle AOB = 110°.

(a) Work out the size of angle APB. Give reasons. **(2)**

(b) Work out the size of angle PAB. Give a reason. **(2)**

(c) Work out the size of angle OAB. **(1)**



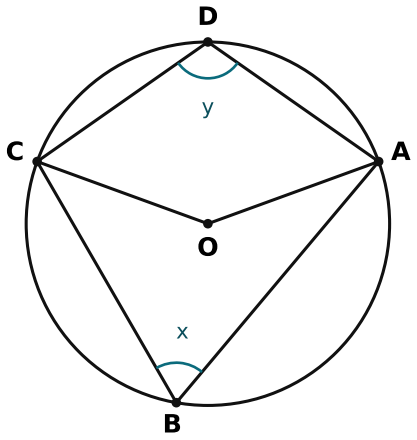

Working (give a reason for every angle)

Answer: _____

**CW13** Proof: cyclic quadrilateral — Grade 9 — Non-calculator — **[5 marks]**

A, B, C and D are points on a circle with centre O. Angle ABC = x and angle ADC = y. Prove that x + y = 180°.

**(5)**




Working (give a reason for every angle)

Answer: _____

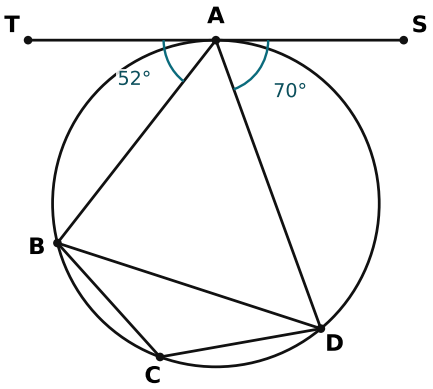
**CW14** Tangent, cyclic quadrilateral and alternate segment — Grade 8 — Non-calculator — **[6 marks]**

TAS is a tangent to the circle at A. B, C and D are points on the circle. Angle TAB = 52° and angle SAD = 70°.

(a) Work out the size of angle BAD. Give a reason. **(2)**

(b) Work out the size of angle BCD. Give a reason. **(2)**

(c) Work out the size of angle ADB. Give a reason. **(2)**



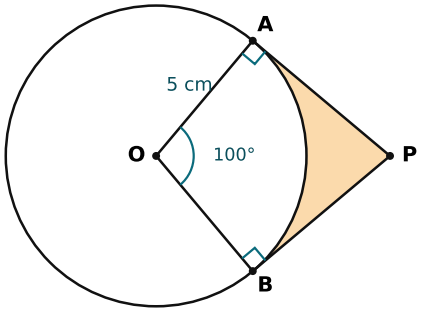

Working (give a reason for every angle)

Answer: _____

**CW15** Tangents with areas Grade 9 Calculator **[6 marks]**

PA and PB are tangents to a circle with centre O and radius 5 cm. Angle AOB = 100°. Work out the area of the shaded region, bounded by PA, PB and the minor arc AB. Give your answer correct to 3 significant figures.

**(6)**




Working (give a reason for every angle)

Answer: _____

## Classwork score tracker

| Q | Topic | Marks | Score |
|---|---|---|---|
| CW1 | Sectors | 4 | |
| CW2 | Angle at the centre | 2 | |
| CW3 | Angle in a semicircle | 2 | |
| CW4 | Cyclic quadrilateral | 4 | |
| CW5 | Angles in the same segment | 3 | |
| CW6 | Tangent and radius | 3 | |
| CW7 | Two tangents | 3 | |
| CW8 | Alternate segment theorem | 4 | |
| CW9 | Chords | 3 | |
| CW10 | Centre theorem and isosceles triangles | 4 | |
| CW11 | Semicircle and cyclic quadrilateral | 4 | |
| CW12 | Tangents and the kite | 5 | |
| CW13 | Proof: cyclic quadrilateral | 5 | |
| CW14 | Tangent, cyclic quadrilateral and alternate segment | 6 | |
| CW15 | Tangents with areas | 6 | |
| **Total** | | **58** | |

Target for grade 9: 80% or more on the homework with every reason written. Below 60%, redo the worked examples of the weakest theorem before moving on.

# Homework

30 questions, 124 marks. Complete in three sets of ten over the week. Questions marked non-calculator must be done by hand.

**HW1** Circle measures — Grade 4 — Non-calculator — **[2 marks]**

The circumference of a circle is 18π cm. Work out the area of the circle. Give your answer in terms of π.

**(2)**

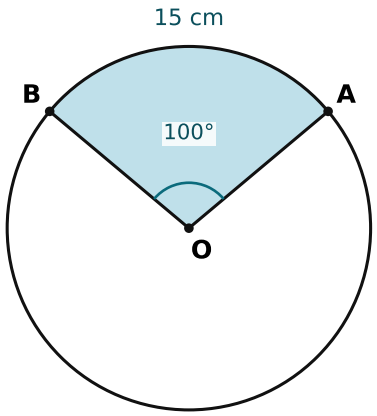
Working (give a reason for every angle)

Answer: _____

**HW2** Sectors — Grade 5 — Calculator — **[3 marks]**

The diagram shows a sector of a circle. The angle at the centre is 100° and the arc length is 15 cm. Work out the radius of the circle. Give your answer correct to 3 significant figures.

**(3)**



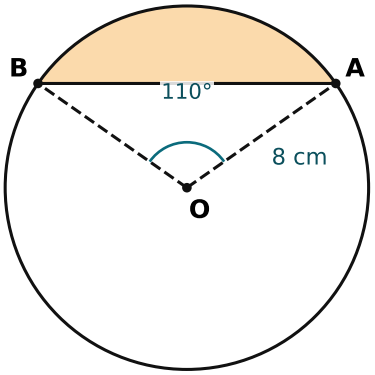

Working (give a reason for every angle)

Answer: _____

**HW3** Segments Grade 7 Calculator **[4 marks]**

The diagram shows a circle of radius 8 cm with centre O. The chord AB subtends an angle of 110° at O. Work out the area of the shaded segment. Give your answer correct to 3 significant figures.

**(4)**



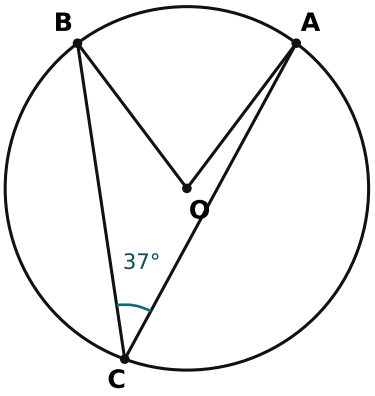

Working (give a reason for every angle)

Answer: _____

**HW4** Angle at the centre Grade 4 Non-calculator **[2 marks]**

A, B and C are points on a circle with centre O. Angle ACB = 37°. Work out the size of angle AOB. Give a reason.

**(2)**



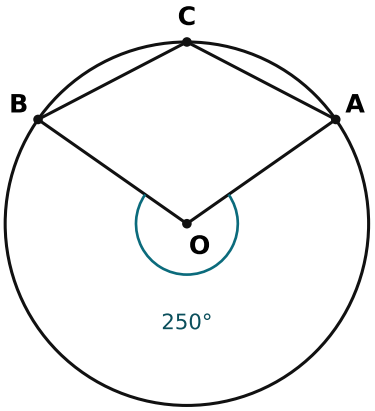

Working (give a reason for every angle)

Answer: _____

**HW5** Angle at the centre (reflex) Grade 7 Non-calculator **[3 marks]**

A, B and C are points on a circle with centre O. C lies on the minor arc AB. The reflex angle AOB is 250°. Work out the size of angle ACB. Give a reason.

**(3)**



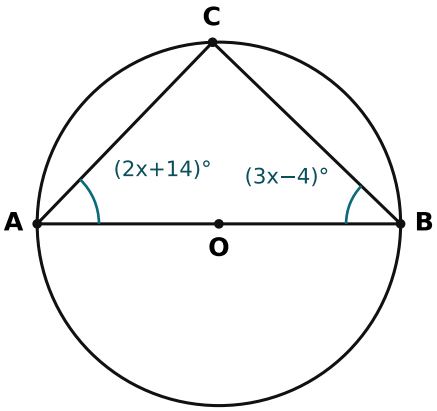

Working (give a reason for every angle)

Answer: _____

**HW6** Angle in a semicircle Grade 6 Non-calculator **[3 marks]**

AB is a diameter of a circle with centre O. C is a point on the circle. Angle BAC = (2x + 14)° and angle ABC = (3x − 4)°. Work out the value of x. Give a reason.

**(3)**




Working (give a reason for every angle)

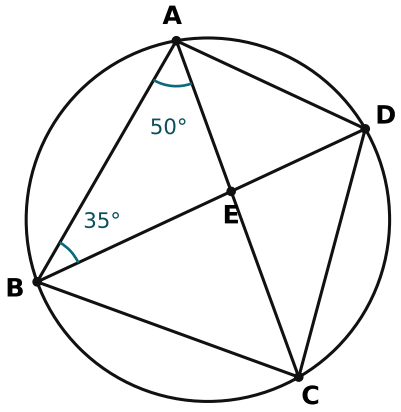
Answer: _____

**HW7** Angles in the same segment — Grade 6 — Non-calculator — **[4 marks]**

A, B, C and D are points on a circle. The chords AC and BD intersect at E. Angle ABD = 35° and angle BAC = 50°.

(a) Write down the size of angle ACD. Give a reason. **(2)**

(b) Work out the size of angle AEB. **(2)**




Working (give a reason for every angle)

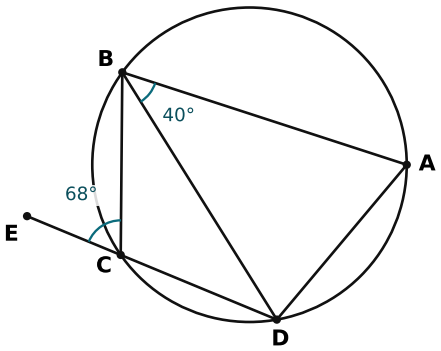
Answer: _____

**HW8** Cyclic quadrilateral (exterior angle) — Grade 6 — Non-calculator — **[3 marks]**

ABCD is a cyclic quadrilateral. DC is extended to E. Angle BCE = 68° and angle ABD = 40°.

(a) Work out the size of angle BAD. Give a reason. **(2)**

(b) Work out the size of angle ADB. **(1)**




Working (give a reason for every angle)

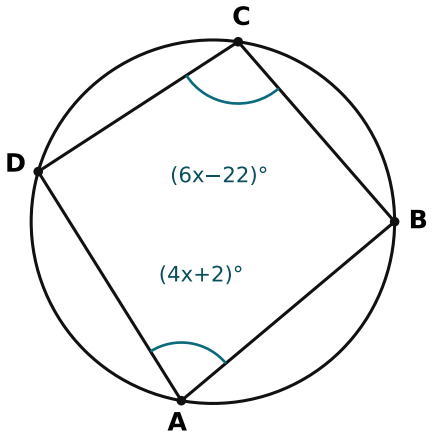
Answer: _____

**HW9** Cyclic quadrilateral (algebra) Grade 7 Non-calculator **[4 marks]**

A, B, C and D are points on a circle. Angle DAB = (4x + 2)° and angle BCD = (6x − 22)°.

(a) Work out the value of x. Give a reason for your first step. **(3)**

(b) Write down the size of angle DAB. **(1)**




Working (give a reason for every angle)

Answer: _____

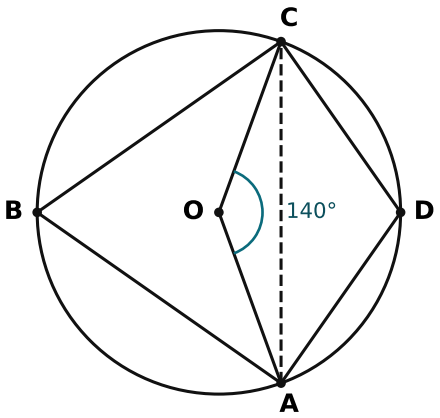
**HW10** Centre theorem and cyclic quadrilateral Grade 8 Non-calculator **[5 marks]**

A, B, C and D are points on a circle with centre O. Angle AOC = 140°. B is on the major arc AC and D is on the minor arc AC.

(a) Work out the size of angle ABC. Give a reason. **(2)**

(b) Work out the size of angle ADC. Give a reason. **(2)**

(c) Work out the size of angle OCA. **(1)**



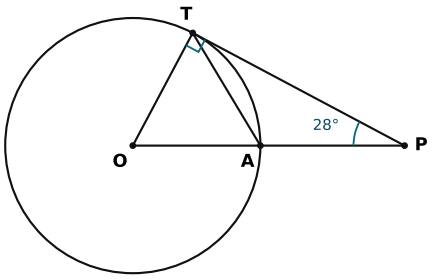

Working (give a reason for every angle)

Answer: _____

**HW11** Tangent and isosceles triangle — Grade 8 — Non-calculator — **[4 marks]**

PT is a tangent to a circle with centre O at the point T. The line PO meets the circle at A. Angle TPO = 28°. Work out the size of angle ATP. You must show all your working.

**(4)**



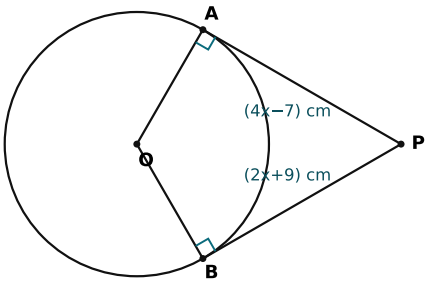

Working (give a reason for every angle)

Answer: _____

**HW12** Two tangents (algebra) — Grade 7 — Non-calculator — **[3 marks]**

PA and PB are tangents to a circle. PA = (4x − 7) cm and PB = (2x + 9) cm. Work out the length of PA. Give a reason for your first step.

**(3)**




Working (give a reason for every angle)

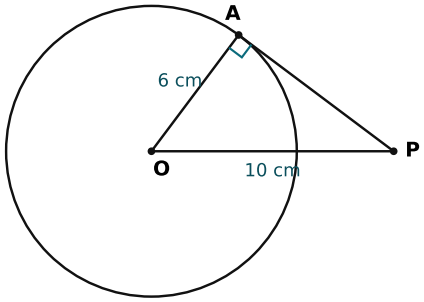
Answer: _____

**HW13** Tangent with Pythagoras and trigonometry — Grade 7 — Calculator — **[4 marks]**

PA is a tangent to a circle with centre O at the point A. The radius of the circle is 6 cm and OP = 10 cm.

(a) Work out the length of PA. **(2)**

(b) Work out the size of angle APO. Give your answer correct to 1 decimal place. **(2)**




Working (give a reason for every angle)

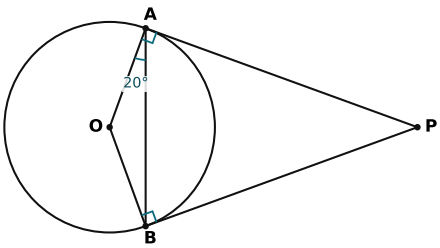
Answer: _____

**HW14** Tangents and isosceles triangles — Grade 8 — Non-calculator — **[5 marks]**

PA and PB are tangents to a circle with centre O. Angle OAB = 20°.

(a) Work out the size of angle APB. Give reasons. **(3)**

(b) Work out the size of angle PAB. **(2)**




Working (give a reason for every angle)

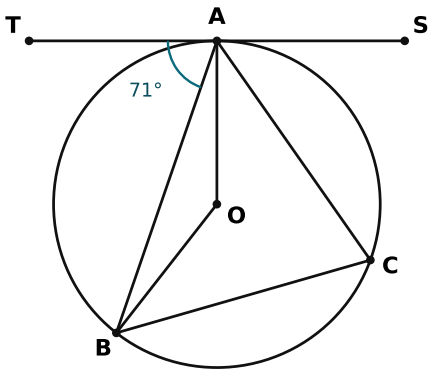
Answer: _____

**HW15** Alternate segment and centre theorem — Grade 7 — Non-calculator — **[3 marks]**

TAS is a tangent to a circle with centre O. A, B and C are points on the circle. Angle TAB = 71°.

(a) Write down the size of angle ACB. Give a reason. **(1)**

(b) Work out the size of angle AOB. Give a reason. **(2)**




Working (give a reason for every angle)

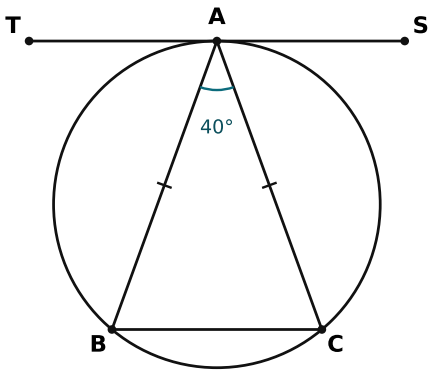
Answer: _____

**HW16** Alternate segment and isosceles triangle — Grade 8 — Non-calculator — **[4 marks]**

A, B and C are points on a circle. AB = AC and angle BAC = 40°. TAS is the tangent to the circle at A.

(a) Work out the size of angle ABC. **(2)**

(b) Work out the size of angle TAB. Give a reason. **(2)**




Working (give a reason for every angle)

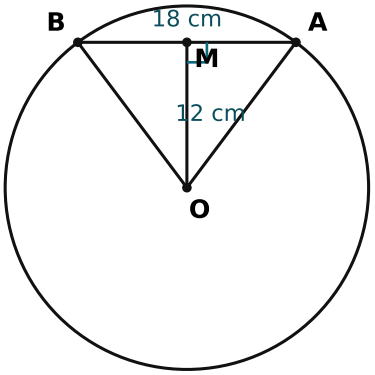
Answer: _____

**HW17** Chords and the centre — Grade 8 — Calculator — **[4 marks]**

AB is a chord of a circle with centre O. M is the midpoint of AB. AB = 18 cm and OM = 12 cm.

(a) Work out the radius of the circle. **(2)**

(b) Work out the size of angle AOB. Give your answer correct to 1 decimal place. **(2)**

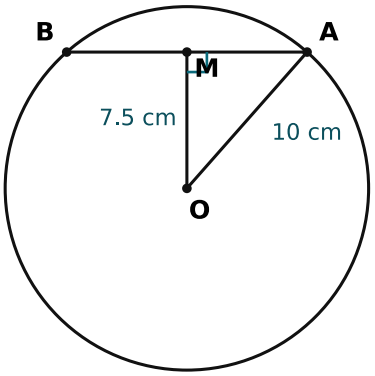



Working (give a reason for every angle)

Answer: _____

**HW18** Chords and Pythagoras — Grade 7 — Calculator — **[3 marks]**

A chord AB is drawn in a circle with centre O and radius 10 cm. The perpendicular distance from O to AB is 7.5 cm. Work out the length of AB. Give your answer correct to 3 significant figures. **(3)**




Working (give a reason for every angle)

Answer: _____

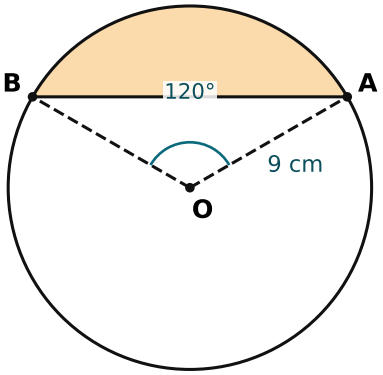
**HW19** Segments — Grade 8 — Calculator — **[5 marks]**

AB is a chord of a circle with centre O and radius 9 cm. Angle AOB = 120°. The shaded region is a segment.

(a) Work out the area of sector OAB, in cm². Give your answer to 3 significant figures. **(2)**

(b) Work out the area of triangle OAB. **(2)**

(c) Work out the area of the shaded segment. **(1)**




Working (give a reason for every angle)

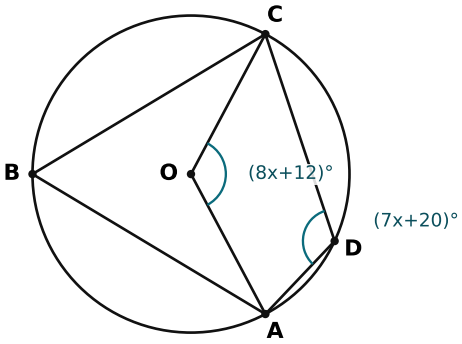
Answer: _____

**HW20** Algebra with several theorems — Grade 9 — Non-calculator — **[6 marks]**

A, B, C and D are points on a circle with centre O. B is on the major arc AC and D is on the minor arc AC. Angle AOC = (8x + 12)° and angle ADC = (7x + 20)°.

(a) Work out the value of x. Give reasons for each step. **(4)**

(b) Work out the size of the reflex angle AOC. **(2)**




Working (give a reason for every angle)

Answer: _____

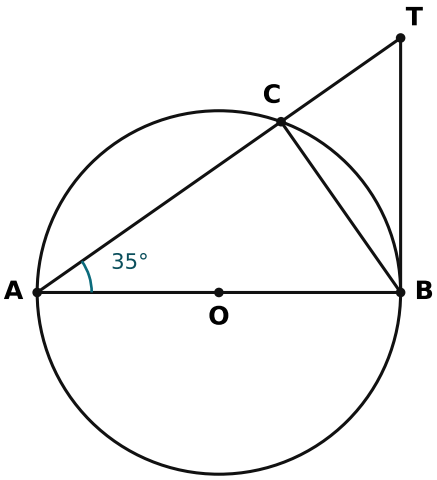
**HW21** Tangent and semicircle — Grade 8 — Non-calculator — **[5 marks]**

AB is a diameter of a circle with centre O. C is a point on the circle. BT is a tangent to the circle at B. AC extended meets BT at T. Angle BAC = 35°.

(a) Write down the size of angle ACB. Give a reason. **(1)**

(b) Work out the size of angle CBT. Give reasons. **(2)**

(c) Work out the size of angle ATB. **(2)**



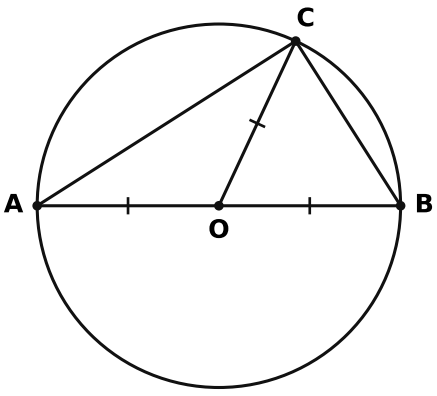

Working (give a reason for every angle)

Answer: _____

**HW22** Proof: angle in a semicircle — Grade 8 — Non-calculator — **[3 marks]**

A, B and C are points on a circle with centre O. AB is a diameter. Prove that angle ACB = 90°.

**(3)**



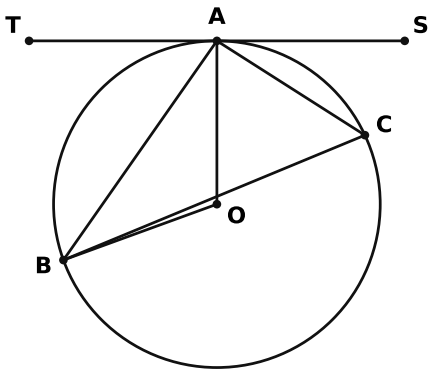

Working (give a reason for every angle)

Answer: _____

**HW23** Proof: alternate segment theorem — Grade 9 — Non-calculator — **[6 marks]**

TAS is a tangent to a circle with centre O at A. B and C are points on the circle. Prove that angle TAB = angle ACB. (You may add a construction to the diagram.)

**(6)**



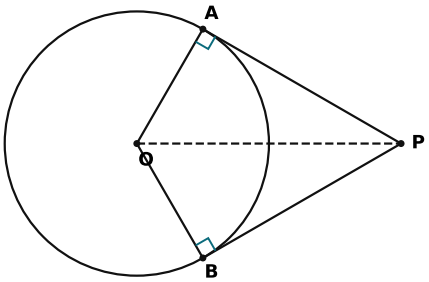

Working (give a reason for every angle)

Answer: _____

**HW24** Proof: equal tangents — Grade 8 — Non-calculator — **[4 marks]**

PA and PB are tangents to a circle with centre O. Prove that PA = PB.

**(4)**




Working (give a reason for every angle)

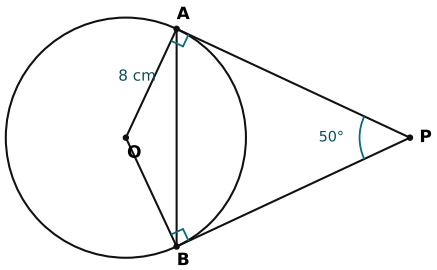
Answer: _____

**HW25** Tangents and trigonometry — Grade 9 — Calculator — **[5 marks]**

PA and PB are tangents to a circle with centre O and radius 8 cm. Angle APB = 50°.

(a) Work out the size of angle AOB. Give reasons. **(2)**

(b) Work out the length of the chord AB. Give your answer correct to 3 significant figures. **(3)**



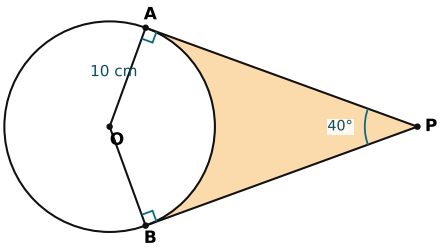

Working (give a reason for every angle)

Answer: _____

**HW26** Tangents with areas — Grade 9 — Calculator — **[6 marks]**

PA and PB are tangents to a circle with centre O and radius 10 cm. Angle APB = 40°. Work out the area of the shaded region, bounded by PA, PB and the minor arc AB. Give your answer correct to 3 significant figures.

**(6)**




Working (give a reason for every angle)

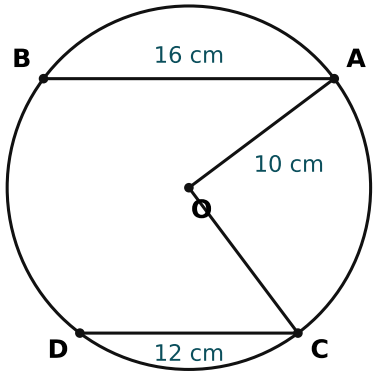
Answer: _____

## HW27 Parallel chords

Grade 8 Non-calculator **[4 marks]**

AB and CD are parallel chords on opposite sides of the centre O of a circle of radius 10 cm. AB = 16 cm and CD = 12 cm. Work out the perpendicular distance between the chords.

**(4)**




Working (give a reason for every angle)

Answer: _____

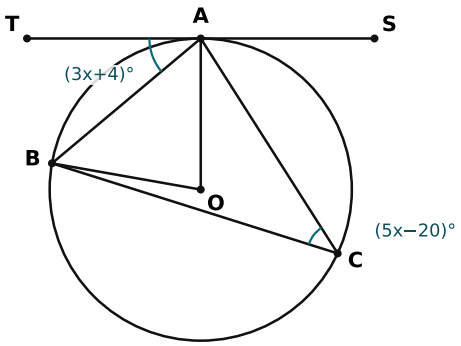
## HW28 Alternate segment (algebra)

Grade 8 Non-calculator **[5 marks]**

TAS is a tangent to a circle with centre O at A. Angle TAB = (3x + 4)° and angle ACB = (5x − 20)°.

(a) Work out the value of x. Give a reason. **(3)**

(b) Work out the size of angle AOB. Give a reason. **(2)**




Working (give a reason for every angle)

Answer: _____

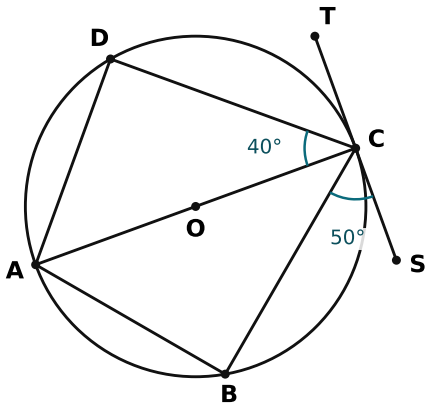
**HW29** Mixed: tangent, semicircle, cyclic quadrilateral Grade 9 Non-calculator **[6 marks]**

AC is a diameter of a circle with centre O. A, B, C and D are points on the circle. ST is the tangent to the circle at C. Angle SCB = 50° and angle ACD = 40°.

(a) Work out the size of angle BAC. Give a reason. **(2)**

(b) Work out the size of angle CAD. Give reasons. **(2)**

(c) Work out the size of angle BCD. Give a reason. **(2)**




Working (give a reason for every angle)

Answer: _____

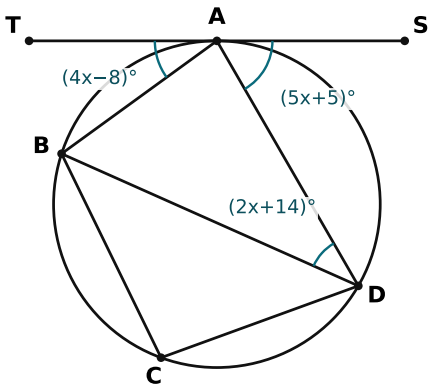
**HW30** Mixed: tangent, algebra, cyclic quadrilateral Grade 9 Non-calculator **[6 marks]**

TAS is a tangent to the circle at A. B, C and D are points on the circle. Angle TAB = (4x − 8)°, angle ADB = (2x + 14)° and angle SAD = (5x + 5)°.

(a) Work out the value of x. Give a reason. **(3)**

(b) Work out the size of angle BAD. **(1)**

(c) Work out the size of angle BCD. Give a reason. **(2)**




Working (give a reason for every angle)

Answer: _____

## Homework score tracker

| Q | Topic | Marks | Score |
|---|---|---|---|
| HW1 | Circle measures | 2 | |
| HW2 | Sectors | 3 | |
| HW3 | Segments | 4 | |
| HW4 | Angle at the centre | 2 | |
| HW5 | Angle at the centre (reflex) | 3 | |
| HW6 | Angle in a semicircle | 3 | |
| HW7 | Angles in the same segment | 4 | |
| HW8 | Cyclic quadrilateral (exterior angle) | 3 | |
| HW9 | Cyclic quadrilateral (algebra) | 4 | |
| HW10 | Centre theorem and cyclic quadrilateral | 5 | |
| HW11 | Tangent and isosceles triangle | 4 | |
| HW12 | Two tangents (algebra) | 3 | |
| HW13 | Tangent with Pythagoras and trigonometry | 4 | |
| HW14 | Tangents and isosceles triangles | 5 | |
| HW15 | Alternate segment and centre theorem | 3 | |
| HW16 | Alternate segment and isosceles triangle | 4 | |
| HW17 | Chords and the centre | 4 | |
| HW18 | Chords and Pythagoras | 3 | |
| HW19 | Segments | 5 | |
| HW20 | Algebra with several theorems | 6 | |
| HW21 | Tangent and semicircle | 5 | |
| HW22 | Proof: angle in a semicircle | 3 | |
| HW23 | Proof: alternate segment theorem | 6 | |
| HW24 | Proof: equal tangents | 4 | |
| HW25 | Tangents and trigonometry | 5 | |
| HW26 | Tangents with areas | 6 | |

| | | | |
|---|---|---|---|
| HW27 | Parallel chords | 4 | |
| HW28 | Alternate segment (algebra) | 5 | |
| HW29 | Mixed: tangent, semicircle, cyclic quadrilateral | 6 | |
| HW30 | Mixed: tangent, algebra, cyclic quadrilateral | 6 | |
| **Total** | | **124** | |

Target for grade 9: 80% or more on the homework with every reason written. Below 60%, redo the worked examples of the weakest theorem before moving on.